Informal Insurance and Agricultural Input Use

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Abstract

When work effort is imperfectly monitored, informal insurance may induce rural households to shirk. Since agricultural inputs have different degrees of complementarity and substitutability with effort, risk-sharing can affect input use through its discouraging effect on effort supply. I characterize the constrained-efficient allocation in a model of risk-sharing where effort and agricultural inputs are private. Insurance induces households to shirk, lowers the use of inputs complementing effort, and boosts the use of inputs substituting effort. Using the last (2009-2014) ICRISAT panel from rural India, I confirm the main theoretical predictions of the model and structurally estimate it. The estimates show that the impact of risk-sharing on fertilizer use is quantitatively important: going from no sharing to full insurance, fertilizer use drops by almost 50% percent.

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1 Introduction

Households in rural economies cope with income risk through a variety of informal insurance (risk-sharing) arrangements, such as gift exchange and informal loans (see [Fafchamps, 2011] for a review). Since the contributions of [Cochrane, 1991], [Mace, 1991], and [Townsend, 1994], it is well known that risk-sharing does not generally achieve perfect consumption smoothing: household income is significant in explaining household consumption. Triggered by empirical evidence rejecting the null hypothesis of full insurance, economists started to think about possible impediments to risk-sharing, focusing on frictions such as limited commitment ([Ligon et al., 2002]), hidden income ([Kinnan, 2017]), and private effort ([Ligon, 1998]).

In this paper, I analyze the effect of risk-sharing on agricultural input use in a village economy subject to private information frictions in production decisions. Private effort has been shown to be a relevant barrier to risk-sharing both in rich and poor countries.[1] The intuition is that when work effort is imperfectly monitored, insurance induces households to shirk. I argue that if informal insurance has a discouraging effect on effort supply, then risk-sharing affects agricultural input use. The technology adoption literature has consistently shown that the use of agricultural inputs is linked to effort supply through relations of complementarity and substitutability.[2] Since agricultural inputs have different degrees of complementarity and substitutability with effort, risk-sharing may relate to different patterns of agricultural input use through its discouraging effect on households' effort supply.

I outline a model of risk-sharing with private effort and private agricultural inputs. Households insure themselves by sharing the profits of agricultural production. Each household can exert (costly) effort and purchase agricultural inputs to increase its mean income. Insurance induces underprovision of effort, as it reduces its private marginal benefit. Effort displays different degrees of complementarity and substitutability with agricultural inputs. As insurance is detrimental effort provision, higher risk-sharing lowers the use of inputs complementing effort

[1] See [Ligon, 1998] for evidence from rural India; [Paulson et al., 2006] and [Karaivanov and Townsend, 2014] for evidence from rural and semi-urban Thailand; [Kocherlakota and Pistaferri, 2009] for evidence from Italy, the UK, and the USA; and [Attanasio and Pavoni, 2011] for evidence from the UK.

[2] For example, it is well-established that effort and fertilizer are complements. See [Foster and Rosenzweig, 2009], [Foster and Rosenzweig, 2010], and [Foster and Rosenzweig, 2011] for evidence from India; [Beaman et al., 2013] for evidence from Mali; [Ricker-Gilbert, 2014] for evidence from Malawi; and [Haider et al., 2018] for evidence from Burkina Faso.

and boosts the use for inputs substituting effort.

I test the model empirically using the last (2009-2014) ICRISAT panel from rural India. First, I provide reduced-form evidence about the main predictions of the model. I show that (i) the key prediction of a model of risk-sharing with private effort is borne out in the data, as insurance is negatively correlated with effort provision; (ii) effort provision and fertilizer use are strongly positively correlated, suggesting the existence of a complementarity between the two inputs; and (iii) as indicated by the theory, insurance is negatively correlated with fertilizer use. While it does not speak to causality, this evidence shows that insurance is an important factor in explaining effort supply and fertilizer use. Consequently, I structurally estimate the model. This estimation allows me to quantitatively assess the role of risk-sharing in effort supply and fertilizer use and conduct counter-factual policy analyses. I find that the impact of risk-sharing on effort supply and fertilizer use is quantitatively important: going from no sharing to full insurance, effort supply decreases by more than six times and fertilizer use drops by almost 50%.

Related literature

This paper studies the joint determination of risk-sharing and agricultural input use in village economies. Insurance is widely believed to play an important role in economic development. While some have argued that a lack of insurance can hold households back from adopting high-risk and high-return technologies ([Rosenzweig and Binswanger, 1993] and [Dercon and Christiaensen, 2011]) others have pointed out that social pressure to share income reduces investment incentives ([Jakiela and Ozier, 2015]). This paper analyzes a new mechanism which relates informal insurance to different patters of agricultural input use through households' effort supply decisions. A novelty of this mechanism is that it allows insurance to have opposite effects on the demand for different inputs, boosting it for inputs that complement effort while lowering it for inputs that substitute effort.

I contribute to the understanding of agricultural input use in low-income countries and, in particular, of how it relates to risk-sharing. Uncovering the determinants of agricultural input use is extremely important, from both an academic and a policy perspective ([Feder et al., 1985], [Sunding and Zilberman, 2001], [Foster and Rosenzweig, 2010], [Udry, 2010], and [Jack, 2013]). It has been argued before that when analyzing the drivers of technology adoption and agricultural input use, it is important to understand the impact of risk-sharing, as it is a pervasive institution of village economies.[3] However, few papers have been written on this topic ([Giné and Yang, 2009] and [Dercon and Christiaensen, 2011]). The only exceptions analyze the case of new technologies, the use of which is discouraged by their uncertain benefits and costs. In this case, better insurance should be associated with higher take-up rates. This paper takes a very different approach, as it focuses on the discouraging effect of insurance on effort supply and how this relates to the use of different agricultural inputs.

The mechanism I propose to link risk-sharing to agricultural input use relies on a private effort friction. The existence of this friction is a key assumption of most of the literature on sharecropping ([Quibria and Rashid, 1984], [Singh, 1991], and [Sen, 2016]). Moreover, private effort has been used to explain imperfect insurance in village economies ([Ligon, 1998]). While several papers have provided evidence for private effort by testing models of imperfect insurance against each other (see [Ligon, 1998], [Ábrahám and Pavoni, 2005], [Kaplan, 2006], [Attanasio and Pavoni, 2011], and [Karaivanov and Townsend, 2014]), this friction has been considered hard to detect using observational data ([Foster and Rosenzweig, 2001]).[4] I contribute to this literature by providing a first *direct* evidence of a negative relationship between insurance and effort.[5] By doing so, I confirm the main implication of the private effort explanation to imperfect insurance.

I embed an agricultural household model ([Singh et al., 1986]), one of the workhorses for modeling technology adoption in village economies, into an informal insurance framework. According to [Foster and Rosenzweig, 2010], studies on input use need to take into account the complementarity and substitutability between inputs, and in particular between labor and agricultural intermediates. Indeed, empirical evidence ([Dorfman, 1996] and [Hornbeck and Naidu, 2014])

[3] According to [Udry, 2010], understanding "how [...] imperfect insurance influence input choice and/or technology adoption in agriculture" is "a key research agenda" in agricultural and development economics.

[4] Despite this shortcoming, there exists experimental evidence of the effect of private effort on risk-sharing ([Prachi, 2016]).

[5] The literature on sharecropping has produced consistent evidence that better risk-sharing (in the form of a lower fraction of the agricultural output going to the tenant) leads to lower efficiency and effort provision ([Laffont and Matoussi, 1995]). However, the same empirical evidence has not been provided by the literature on informal insurance.

suggests that labor availability plays a key role in the decision to adopt different input baskets. By taking into account the complementarity and substitutability between effort and other inputs, my model directly speaks to this issue, as it explicitly recognizes that the profitability of an input (and hence its use) will ultimately depend on the households' willingness to allocate their time to farm labor (which is in turn affected by how insured they are).

Finally, this paper relates to a growing literature focusing on how informal insurance affects different economic aspects of the village economy. Important contributions to this literature are [Munshi and Rosenzweig, 2006], which studies how risk-sharing shapes career choice by gender in Bombay; [Munshi and Rosenzweig, 2016], which analyzes how caste-based informal insurance affects incentives to migrate in India; [Advani, 2017], which studies how informal insurance with limited commitment impacts on investment in livestock in Bangladesh; and [Morten, 2017], which studies the joint determination of informal insurance and temporary migration in rural India when there is a limited commitment friction.

The rest of the paper is organized as follows. In Section 2, I outline a simple model of exogenous risk-sharing and conduct comparative statics exercises to generate testable implications. In Section C, I extent the model to allow for endogenous risk-sharing and show that the main predictions of the model outlined in 2 carry through. In Section 3, I introduce the data, provide reduced-form evidence confirming the main implications of the model, and structurally estimate the simple model of exogenous risk-sharing outlined in Section 2. Finally, Section 4 concludes and points to future research.

2 A Simple Model of Risk-Sharing

In this section, I outline a static model of risk-sharing with private effort and private agricultural inputs. Insurance is modeled in a stylized way by only considering linear sharing contracts.[6] The results obtained in this section do not rely on these two stringent assumptions, as shown in Appendix C, which extends the model to endogenous risk-sharing. All the proofs are contained

[6]Linear sharing contracts are not generally optimal when choices of effort and intermediates are private. Yet, linearity can be motivated by empirical evidence, as in [Dutta and Prasad, 2002], and simplifies the analysis considerably.

in Appendix A.

There are n household-farms, each producing output y_i, $i \in N := \{1, \ldots, n\}$. Output is uncertain, and depends on effort $e_i \in [0, \overline{e}_i]$ and quantities of agricultural inputs (intermediates) $\boldsymbol{z}_i \in \mathbb{R}_+^m$. In particular,

$$y_i := y\left(e_i, \boldsymbol{z}_i\right) + \varepsilon_i, \tag{1}$$

where y is strictly increasing, strictly concave, and twice-continuously differentiable in all arguments, as well as jointly concave in $(e_i, \boldsymbol{z}_i)$, and ε_i is a production shock with mean μ and variance Σ^2. Hence, supplying more effort or increasing the use of an agricultural input improves the expectation of output without making it riskier. The shocks are independently distributed across households. Letting $\boldsymbol{p}$ be a vector of prices for the agricultural intermediates, household i's agricultural profit (income) is given by

$$\pi_i = y_i - \boldsymbol{p} \cdot \boldsymbol{z}_i. \tag{2}$$

Households share incomes[7] to smooth consumption risk. In particular, household i's consumption is given by

$$c_i\left(\alpha\right) = \left(1 - \alpha\right) \pi_i + \alpha \overline{\pi}, \tag{3}$$

where $\alpha \in [0, 1]$ is a parameter that fully characterizes the extent of risk-sharing and $\overline{\pi}$ is average income. The intuition is that each household consumes a fraction $1 - \alpha$ of its agricultural profit, and contributes the rest to a common pool which is shared equally.[8] Risk-sharing is assumed to be enforceable.

[7]Given that income is defined as agricultural profit, households share both production and the monetary cost of agricultural inputs. This assumption plays an important role, as it implies that risk-sharing only has a direct impact on effort choices. Of course in equilibrium risk-sharing does affect agricultural input use, but this effect only comes about through its impact on effort. While the informal insurance literature has maybe accustomated us to think of households sharing outputs, this is mainly an artifact due to the fact that most models abstract away from the presence of inputs purchased in the market. Moreover, the assumption that household income equals agricultural profit is consistent with the way in which farm household income is measured in practice ([The Organisation for Economic Co-operation and Development, 2003]).

[8]Equation (3) can be thought of as an implementation of the well-known contrast estimator ([Suri, 2005]) when the economy is closed and there are no saving technologies, as shown in Appendix B.

Household i's expected utility function is given by[9]

$$U\left(c_i\left(\alpha\right), e_i\right) := \mathbb{E}\left(c_i\left(\alpha\right)\right) - \frac{\rho}{2}\mathbb{V}\mathrm{ar}\left(c_i\left(\alpha\right)\right) - \kappa e_i,$$

where ρ is the coefficient of absolute risk aversion and κ is the marginal disutility of effort. Expectations are taken with respect to the distribution of $c_i\left(\alpha\right)$ conditional on e_i.

I begin the analysis by fixing some α. Doing so allows me to draw the mapping from sharing rules to household choices. Once this mapping is drawn, the planner can take it into account to derive how sharing rules map to social welfare, hence solving his problem. When α is fixed, an allocation is simply a profile of efforts and agricultural intermediates $(\boldsymbol{e}, \boldsymbol{z}) := (e_i, \boldsymbol{z}_i)_{i\in N}$. Given Pareto weights λ_i, the planner's problem under full information is

$$\max_{\boldsymbol{e},\boldsymbol{z}} \sum_{i\in N} \lambda_i U\left(c_i\left(\alpha\right), e_i\right), \tag{4}$$

subject to Equations (3), (2), and (1). When effort and agricultural inputs are private information, the planner's problem is

$$\begin{aligned} &\max_{\boldsymbol{e},\boldsymbol{z}} \sum_{i\in N} \lambda_i U\left(c_i\left(\alpha\right), e_i\right), \\ &\text{subject to } \left(e_i, \boldsymbol{z}_i\right) \in \arg\max_{\widehat{e}_i} U\left(c_i\left(\alpha\right), \widehat{e}_i\right) \end{aligned} \tag{5}$$

and Equations (3), (2), and (1).

Claim 1 (Efficient allocation)**.** *The solution to the planner's problem under full information is characterized by*

$$y_e\left(e_i^*, \boldsymbol{z}_i^*\right) = \kappa,$$

$$y_{\boldsymbol{z}}\left(e_i^*, \boldsymbol{z}_i^*\right) = \boldsymbol{p}.$$

[9] I make use of a mean-variance expected utility of consumption because it greatly simplifies strategic interactions between households: given some α, i's choices of effort and intermediates do not depend on the other households' choices. See Subsection C.1 for a more detailed discussion.

Claim 2 (Constrained-efficient allocation)**.** *The solution to the planner's problem under private information is characterized by*

$$y_e\left(e_i^*, \boldsymbol{z}_i^*\right) = \frac{\kappa}{\left(1 - \frac{n-1}{n}\alpha\right)} =: p_e\left(\alpha\right),$$
$$y_{\boldsymbol{z}}\left(e_i^*, \boldsymbol{z}_i^*\right) = \boldsymbol{p},$$

for each $i \in N$.

Claim 2 shows that risk-sharing affects the 'effective price' of effort, p_e, but not the prices of the agricultural intermediates. Next, I show main theorem relating risk-sharing to choices of effort and agricultural intermediates.

Theorem 1 (Comparative statics)**.** *Let $(e^*, \boldsymbol{z}^*)$ be the second best allocation. Then,*

$$\frac{\partial e_i{}^*\left(p_e, \boldsymbol{p}\right)}{\partial \alpha} < 0.$$

Moreover, suppose that e_i and z_i^q are complements at $(p_e, \boldsymbol{p})$; i.e.,

$$\frac{\partial z_i^{q*}\left(p_e, \boldsymbol{p}\right)}{\partial p_e} < 0.$$

Then,

$$\frac{\partial z_i^{q*}\left(p_e, \boldsymbol{p}\right)}{\partial \alpha} < 0.$$

The signs of the latter two inequalities are reversed if e_i and z_i^q are substitutes at $(p_e, \boldsymbol{p})$.

Theorem 1 shows that if risk-sharing increases, then households exert less effort, increase the use of agricultural intermediates that complement effort, and decrease the use agricultural intermediates that substitute effort.[10]

Given the mapping from sharing rules to household choices implied by Theorem 1, let me consider the problem of finding an optimal sharing contract. To simplify the analysis, I assume

[10] See Appendix D for a discussion on the definition of complementarity and substitutability used in Theorem 1, as well as Theorem 2 in Section C, and other possible definitions of complementarity and substitutability (and additional assumptions on the production function) under which these theorems would still hold.

that the planner places equal weight on all households. The following claim shows that, under full information, risk-sharing is perfect.

Claim 3 (Efficient sharing)**.** *Under full information, the optimal sharing contract is full insurance.*

The next claim shows that, under private information, characterizes an optimal sharing contract under private information, and highlights that, under this information regime, a marginal increase in α generates a trade-off between decreasing consumption volatility and decreasing aggregate consumption.

Claim 4 (Constrained-efficient sharing)**.** *Let*

$$\frac{\partial W(\alpha)}{\partial \alpha} := \underbrace{\sum_{i \in N} \left(\kappa \left(\frac{1}{1 - \frac{n-1}{n}\alpha} - 1 \right) \frac{\partial e_i^*}{\partial \alpha} \right)}_{(-)} \underbrace{- \frac{n\rho}{2} \frac{\partial \mathbb{V}\mathrm{ar}\left(c_i(\alpha)\right)}{\partial \alpha}}_{(+)} \tag{6}$$

and α^* be an optimal sharing rule under private information. Then, it must be the case

$$\begin{aligned} \tfrac{\partial W(\alpha)}{\partial \alpha} &= 0 \quad \textit{if } \alpha \in (0,1), \\ \tfrac{\partial W(\alpha)}{\partial \alpha} &\leq 0 \quad \textit{if } \alpha = 0, \\ \tfrac{\partial W(\alpha)}{\partial \alpha} &\geq 0 \quad \textit{if } \alpha = 1. \end{aligned}$$

The first term of Equation (6), which is negative, is the loss in aggregate produced by a marginal increase in the negative externality caused by sharing. The second term, which is positive, is the gain associated with a marginal reduction in consumption volatility. Hence, in general, one should not expect to observe $\alpha = 1$, as under full information.

3 Empirical Evidence

3.1 Background and Data

I use data collected under the VDSA project by ICRISAT. This is a household-level panel data providing detailed information on farming, expenditure, and income for more than 700 households in 18 villages in the Indian semi-arid tropics. The data comes from survey interviews conducted at a monthly frequency from 2009 to 2014.

For each village, 40 households were randomly selected stratifying by own landholding classes: 10 from landless laborers, 10 from small farmers, 10 from medium farmers, and 10 from large farmers.

There are three problems with the data collected at a monthly frequency: (i) the frequency of the interviews varies, (ii) the interview dates differ across households, and (iii) recall periods vary across interviews. Fortunately, from 2010 onwards, information is provided on the month and year to which a given interview refers to. Since recall periods can be longer than a month, it is impossible to determine to which month an interview refers to if this information is not provided. Therefore, I drop the observations for 2009.

For the estimation, I need information on demographic characteristics, consumption, income, effort, and agricultural intermediates. In the following, I discuss how variables are constructed. Money values are converted in 1975 rupees for comparability with [Townsend, 1994].

Following [Mazzocco, 2012], I use the data coming from the General Endowment Schedule to construct a set of observable household heterogeneity variables, which is comprised of the average age of adult household members, the number of infants, and the age-sex weight proposed in [Townsend, 1994].

Monthly household consumption is calculated using the Transaction Schedule. Data is collected on the value of items purchased, home produced, and acquired in other ways (such as gifts). Following [Kinnan, 2017], I sum the value of all items across categories to construct a measure of total expenditure. Since different households have different sizes and age-sex structure, I convert total expenditure to adult-equivalent terms using the age-sex weight.

Monthly household income is calculated using the methodology proposed in [Mazzocco, 2012]. Making use of the household budget constraint, total income is computed as total expenditure minus resources borrowed, plus resources lent and saved, minus government benefits. Information on these variables is contained in the Transaction Schedule. The data is aggregated following the same procedure I use to calculate monthly household consumption. I convert total income to adult-equivalent terms using the age-sex weight.

Table 1: Summary Statistics

Variable	Average	Std. Dev.
Household size	5.18	2.44
Number of infants	0.04	0.22
Average adult age	41.28	9.06
Age-sex weight	4.51	1.93
Monthly consumption	124.39	82.07
Monthly income	123.11	164.34
Monthly effort	22.55	34.53
Number of households	400	
Observations	9763	

Notes: All money values in 1975 rupees. Consumption, income, and effort expressed in adult-equivalent terms. Effort is hours of work per acre supplied by family members. Household-month observations.

Effort is proxied by the total hours of work supplied by family members to their plots.[11] Monthly household effort and expenditure on agricultural intermediates is calculated from the information provided in the Cultivation Schedule. In this module, information is collected about the quantity and total value of each type of agricultural intermediate and labor used in each operation performed on every plot. A distinction is made between family, hired, and exchanged labor. I take the quantity of family labor supplied to each operation and aggregate this information at the household-month level to compute the total amount of labor (hours) supplied by family members. Then, I convert this measure to adult-equivalent terms using the age-sex weight. The same procedure is utilized on expenditure on several agricultural inputs to generate a measure of monthly per-capita expenditure on each input.

Summary statistics for the sample are reported in Table 1.

3.2 Reduced-Form Evidence Linking Production and Risk-Sharing

I document the following facts in the data: (i) risk-sharing is imperfect; (ii) effort is lower when risk-sharing is better; (iii) effort supply and fertilizer use are positively correlated; and (iv) effort is lower when risk-sharing is better.

[11] Proxing effort by hours of work is a rather standard practice: see e.g. [Clark et al., 2003].

Table 2: A Test for Full Sharing

Dep. variable: $\log(c_{it})$	$\widehat{\beta}$ (s.e.)
$\log(y_{it})$	.2594*** (.0207)
Household fixed effects	Yes
Village-month fixed effects	Yes
R-squared	0.773
Observations	18,931

Notes: OLS regressions of log income on log consumption. Standard errors are robust.

Risk-sharing is imperfect. I estimate the following regression for household i in village v and month t:

$$\log(c_{it}) = \beta_1 \log(y_{it}) + \varphi_i + \phi_{vt} + \epsilon_{it}, \tag{7}$$

where φ_i and ϕ_{vt} are household and village-month fixed effects. Equation (7) is taken from [Morten, 2017]. It estimates the elasticity of consumption with respect to income after controlling for aggregate income through village-month fixed effects. Standard errors are robust.

Table 2 reports the results of the test. Full sharing is strongly rejected. The elasticity of consumption with respect to income is approximately 0.26. Although the magnitude of this coefficient varies across studies, a value of 0.26 falls well within the expected range. For instance, [Munshi and Rosenzweig, 2009] estimate values between 0.17 and 0.26 for rural India, using data from the Rural Economic and Demographic Survey (REDS); [Cochrane, 1991] finds values between 0.1 and 0.2 for the United States using data from the Panel Study of Income Dynamic (PSID); [Milán, 2016] finds a value of 0.35 for indigenous villages in the Bolivian Amazon. Overall, the results square fairly well with the literature and unequivocally reject full insurance.

As robustness checks, I estimate Equation (7) aggregating the data at a quarterly and annual frequency, and run the alternative specifications outlined in [Jalan and Ravallion, 1999] and [Mazzocco, 2012]. Reassuringly, the results do not change.

Effort is lower when risk-sharing is better. Theorem 1 says that effort decreases when risk-sharing increases. To analyze the correlation between risk-sharing and effort, I follow

Table 3: Risk-Sharing and Effort

Dep. variable: $\log(c_{it})$	$\widehat{\beta}$ (s.e.)
$\log(y_{it})$	.1878*** (.0515)
$\log(y_{it}) \times \overline{\log(e_i)}$	.0278* (.0155)
Household fixed effects	Yes
Village-month fixed effects	Yes
R-squared	0.773
Observations	18,929

Notes: OLS regressions of log income and log income times average log effort on log consumption. Standard errors clustered are robust.

[Morten, 2017] and estimate the following regression:

$$\log(c_{it}) = \beta_1 \log(y_{it}) + \beta_2 \log(y_{it}) \overline{\log(e_i)} + \varphi_i + \phi_{vt} + \zeta_{it}, \tag{8}$$

where e_{it} is the adult-equivalent total work hours supplied to own fields by household i in month t, and $\overline{\log(e_i)}$ is the average effort supplied by household i.[12] Coefficient β_2 is the correlation between average effort supplied and the elasticity of consumption with respect to income. If insurance is negatively correlated to effort, β_2 should be positive. In this case, the slope of consumption on income is higher as average effort increases. That is, comparing two households which are identical across any dimension captured by the household and village-month fixed effects, a positive β_2 indicates that the consumption of the household that supplied more effort is expected to be more responsive to idiosyncratic shocks to own income. Table 3 reports the results of the OLS estimation of Equation (8). The interaction term is positive and significant. To get a sense of the magnitudes, assume that effort supply is constant in time. Then, on average, doubling effort provision is associated to 14% increase in the elasticity of consumption with respect to income. This confirms that households that are less well insured put more effort.

Notice that I interact $\log(y_{it})$ with $\overline{\log(e_i)}$ instead of $\overline{e}_i$, hence effectively giving less weight

[12] The main effect of $\overline{\log(e_i)}$ is omitted because it is captured by the household fixed effects.

Table 4: Non-Linearity between Risk-Sharing and Effort

Dep. variable: $\log(c_{it})$	$\widehat{\beta}$ (s.e.)
$\log(y_{it})$	.1841*** (.0416)
$\log(y_{it}) \times \overline{e}_i$	.0048** (.0016)
$\log(y_{it}) \times \overline{e}_i^2$	-.0001** (.0001)
Household fixed effects	Yes
Village-month fixed effects	Yes
R-squared	0.774
Observations	18,931

Notes: OLS regressions of log income, log income times average effort, and log income times average effort squared on log consumption. Standard errors clustered are robust.

to higher values of $\overline{e}_i$. I do this because the relationship between insurance and and effort is non-linear. This can be seen by running the following regression:

$$\log(c_{it}) = \beta_1 \log(y_{it}) + \beta_2 \log(y_{it})\,\overline{e}_i + \beta_3 \log(y_{it})\,\overline{e}_i^2 + \varphi_i + \phi_{vt} + \zeta_{it}. \tag{9}$$

Table 4 gives the results of the OLS estimation of the Equation (9). The interaction between the log of income and average effort is positive and significant, confirming the results obtained in Table 3. However, the interaction between the log of income and average effort squared is negative and significant. This suggests that as average effort supply increases, the negative relationship between effort and insurance becomes weaker. When running regression (9) without the interaction between the log of income and average effort squared, β_2 narrowly becomes insignificant, suggesting that the non-linearity between effort and insurance is important.

One might be concerned that the right measure of effort is not adult-equivalent work hours, but total adult-equivalent work hours per acre;[13] hence, I re-estimate Equation (8) using total

[13] In particular, the productivity of a household is expected to be very unequally distributed. For example, suppose that households with more land are more productive, or they have a higher incentive to use adult-equivalent work hours more productively, so that the marginal product of an adult-equivalent work hour per unit of land is higher for households with more land. In that case, a marginal change in insurance would produce a higher effect on the adult-equivalent work hours per acre supplied by households with less landholdings, and

Table 5: Effort and Fertilizer

Dep. variable: $\log(f_{it})$	$\widehat{\gamma}$ (s.e.)
$\log(e_{it})$	.4348*** (.0141)
Land area	.0032*** (.0003)
Household fixed effects	Yes
Village-month fixed effects	Yes
R-squared	0.326
Observations	8,794

Notes: OLS regressions of effort on fertilizer. All regressions with household and month fixed effects. Standard errors are robust.

work hours per capita and acre as a measure of effort. The results do not change.

Even though these tests do not speak to causality, they are consistent with the model and provide suggestive evidence about the disincentive effect of insurance.

Effort and fertilizer. Next, I provide evidence about the complementary between effort and fertilizer. I run the following regression equation:

$$\log(f_{it}) = \gamma \log(e_{it}) + \varphi_i + \ell_{it} + \tau_t + \nu_{it}, \tag{10}$$

where f_{ivt} is the adult-equivalent *value* of fertilizer used by household i,[14], ℓ_{ivt} is land area, and τ_t are month fixed effects. My measure of fertilizer includes organic compounds (such as urea), micro-nutrients, and manure. The results are reported in Table 5. Fertilizer is generally considered to be a land-augmenting technologies: in efficiency units, a quantity of fertilized land can be conceived as a multiple of a smaller quantity of unfertilized land. Hence, whenever land and effort are complementary, so are effort and fertilizer. Table 5 shows that effort is significantly positively correlated with fertilizer.

I run a version of Equation (10) in levels as robustness check, and find that all the results

part of cross-sectional variation in effort that one observes may be driven by heterogeneity in land holdings.

[14] I focus on value instead of physical quantity for simplicity, as there is heterogeneity in the units of measurement of inputs. However results do not change when I attempt to convert physical quantities to kilograms and aggregate to obtain a measure of total quantity of fertilizer used.

Table 6: Risk-Sharing and Fertilizer

Dep. variable: Consumption	$\widehat{\beta}$ (s.e.)
Income	.0575 (.1014)
Income × fertilizer	.0619** (.0269)
Household fixed effects	Yes
Village-month fixed effects	Yes
R-squared	0.774
Observations	18,890

Notes: OLS regressions of log income and log income times average log fertilizer on log consumption. Standard errors are robust.

go through.

As a cautionary note, these regressions cannot be interpreted in a casual fashion. Despite these issues, the evidence provided strongly confirms the existence of complementarity between effort and fertilizer.

Risk-sharing and fertilizer. Theorem 1 implies that (i) if a technology is complementary to effort, households use less of it as long as they are better insured; (ii) if a technology substitutes effort, households use more of it as long as they are better insured. Before, I provided evidence about the complementarity between effort and fertilizer. Hence, I expect to observe a negative correlation between insurance and fertilizer use. To test this hypothesis, I begin by running the following regressions:

$$\log\left(c_{ivt}\right) = \beta_1 \log\left(y_{ivt}\right) + \beta_3 \log\left(y_{ivt}\right) \overline{\log\left(f_{ivt}\right)} + \varphi_i + \phi_{vt} + \varrho_{ivt}. \tag{11}$$

The correlation between average fertilizer use and the elasticity of consumption with respect to income is given by β_3. Table 6 reports the results of running regression (11). If insurance is negatively correlated to the use of fertilizer, β_3 should be positive, and negative otherwise. Indeed, the results show that β_3 is positive and significant.

For completeness, I test the non-linearity between risk-sharing and fertilizer use by running

Table 7: Non-Linearity between Risk-Sharing and Fertilizer

Dep. variable: $\log(c_{it})$	$\widehat{\beta}$ (s.e.)
$\log(y_{it})$	.2143*** (.0416)
$\log(y_{it}) \times \overline{f}_i$	.0026 (.0022)
$\log(y_{it}) \times \overline{f}_i^2$	-.0001 (.0001)
Household fixed effects	Yes
Village-month fixed effects	Yes
R-squared	0.773
Observations	18,931

Notes: OLS regressions of log income, log income times average fertilizer, and log income times average fertilizer squared on log consumption. Standard errors clustered are robust.

the following regression:

$$\log(c_{it}) = \beta_1 \log(y_{it}) + \beta_2 \log(y_{it}) \overline{f}_i + \beta_3 \log(y_{it}) \overline{f}_i^2 + \varphi_i + \phi_{vt} + \zeta_{it}. \quad (12)$$

Table 7 gives the results of the OLS estimation of the Equation (9). While coefficients β_2 and β_3 narrowly lose significance, the signs of the coefficients clearly confirm the same intuition provided by Table 4.

3.3 Structural Estimation of a Model of Exogenous Risk-Sharing

[PRELIMINARY AND INCOMPLETE]

In this subsection, I estimate the simple model of exogenous risk-sharing outlined in Section 2 and perform comparative statics exercises. The results of this estimation quantify the effect of risk-sharing on effort supply and agricultural input use. The empirical strategy uses the model in Section 2 to draw a map from the distributions of fertilizer price, risk-sharing, and village size (the exogenous variables) to the distributions of effort exerted and fertilizer used (the endogenous variables). I make use of the variation in the exogenous variables to consistently estimate unobserved parameters of the model.[15]

This subsection begins by describing the identification of the model and the estimation procedure. In the model outlined in Section 2, risk-sharing is exogenous, and equilibrium behavior is determined by households' choices of effort and agricultural inputs. The model in Section 2 is particularly easy to estimate, as the strategic interaction between households are greatly simplified by the assumptions that (i) households have mean-variance expected utility and (ii) the sharing contract is linear (see Subsection C.1). Relaxing these assumptions would generally make strategic interactions more complex; in that case, it would be substantially more difficult to solve the model than in the case in which a household's choices of effort and agricultural inputs are independent what the other households do. On the negative side, making assumptions to simplify strategic interactions is detrimental to the richness of the model and its ability to capture relevant sources of variation in the data.

In the data, I observe consumption (c), income (π), effort (e), and fertilizer use (f). The parameters to be identified are the disutility of effort (κ) and the technology parameter (σ) of a production function to be specified. Here, I firstly impose additional restrictions that will allow me to identify these parameters. Then, I explain how I estimate the model.

Letting $\boldsymbol{a}_i := (e_i, f_i)$, assume that the value of agricultural output is given by

$$y\left(\boldsymbol{a}_i\right) = \ell_i^{1-\chi}\left[e_i^{\frac{\sigma-1}{\sigma}} + f_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\chi\sigma}{\sigma-1}},$$

where $\chi \in (0,1)$, $\sigma \in (0,\infty)$ is the elasticity of substitution between effort and fertilizer, and ℓ_i is land, which I assume to be fixed.[16] With this production function, the first-order conditions

[15] The proper estimation of a model of endogenous risk-sharing would require the drawing of a map from the distributions of fertilizer price and village size, and the variance of income shocks to the distributions of effort, fertilizer use, and consumption. Nevertheless, the results presented in this subsection are consistent with a model of endogenous risk-sharing in which the planner is constrained to use linear sharing contracts. In this case, the estimates presented here give the endogenous relationship between risk-sharing and household choices pinned down by the IC constraints; i.e., all the pairs $(\alpha, \boldsymbol{a}_i)$ such that $\boldsymbol{a}_i$ solves problem 25. Given this relationship, a given sharing rule α delivers the implied actions taken by the households, as well as the distribution of consumption through the relationship $c_i(\alpha) = (1-\alpha)\,\pi_i + \alpha\overline{\pi}$. Hence, when solving his problem, the planner just needs to pick a sharing rule while taking into account the relationship between sharing rules and household choices implied by incentive compatibility.

[16] Hence, the production function exhibits decreasing returns to scale in $\boldsymbol{a}_i$. To see why I need decreasing returns, notice that the household's problem is equivalent to that of a competitive firm facing a real price of fertilizer equal to p and a real price of effort equal to $\kappa\left(1+\frac{n-1}{n}\alpha\right)^{-1}$. This is easily checked by considering the problem of such a firm and noticing that the profit-maximizing choices of effort and fertilizer coincide with the first-order conditions for utility maximization given in Claim 2. But under constant returns, the profit-

for effort and fertilizer given in Claim 2 read as follows:

$$\ell_i^{1-\chi}\chi\left[e_i^{*\frac{\sigma-1}{\sigma}}+f_i^{*\frac{\sigma-1}{\sigma}}\right]^{\frac{\chi\sigma}{\sigma-1}-1}e_i^{*-\frac{1}{\sigma}}=\frac{\kappa}{\left(1-\frac{n-1}{n}\alpha\right)}$$

and

$$\ell_i^{1-\chi}\chi\left[e_i^{*\frac{\sigma-1}{\sigma}}+f_i^{*\frac{\sigma-1}{\sigma}}\right]^{\frac{\chi\sigma}{\sigma-1}-1}f_i^{*-\frac{1}{\sigma}}=p.$$

Dividing the second equation by the first one, I get

$$\left(\frac{f_i^*}{e_i^*}\right)^{-\frac{1}{\sigma}}=\frac{p\left(1-\frac{n-1}{n}\alpha\right)}{\kappa}.$$

Rearranging and taking logs, I obtain

$$\log\left(\frac{f_i^*}{e_i^*}\right)=\sigma\log\left(\kappa\right)-\sigma\log\left(1-\frac{n-1}{n}\alpha\right)-\sigma\log\left(p\right).$$

Assuming that the observed choices of effort and fertilizer are optimal, if there is an error in the measurement of fertilizer or effort, I can estimate

$$\log\left(\frac{f_{it}}{e_{it}}\right)=\sigma\log\left(\kappa_i\right)-\sigma\log\left(1-\frac{n_{vt}-1}{n_{vt}}\alpha_{vt}\right)-\sigma\log\left(p_{it}\right)+\epsilon_{it}, \tag{13}$$

where I am assuming that the disutility of effort, κ, is constant in time but possibly heterogeneous across households. Notice that village size and risk-sharing are allowed to be time-varying and village-specific. In principle, α_{vt} could also be defined at the household level. For example, when using the time series estimation proposed by [Townsend, 1994], one essentially computes a time-invariant risk-sharing coefficient for each household. However, if one estimates α by following a pooling strategy (or even using the contrast estimator), as I do in Subsection 3.2, then this coefficient is constant across households, villages, and time. I do not take a stance on the issue of whether α_{vt} varies across time or villages, but I do require α_{vt} *not* to be household-specific. This is a necessary condition to identify the parameters of interest (κ_{iv} and σ), as

maximizing choices of inputs by a competitive firm are indeterminate. So, either one imposes an additional constraint to pin down some *ad hoc* production level, and hence back out $\boldsymbol{a}_i^*$, or one should abandon constant returns to scale.

Table 8: Structural Estimation

Dep. variable: $\log\left(\frac{f_{it}}{e_{it}}\right)$	$\widehat{\beta}$ (s.e.)
$\log(p_{it})$	-.5944** (.3017)
Household fixed effects	Yes
Village-month fixed effects	Yes
R-squared	0.293
Observations	6,481

Notes: OLS regressions of log fertilizer price on log fertilizer over effort. Standard errors are robust.

explained below.

Estimation. Under the premise that the model is correctly specified, the underlying assumptions for the consistent estimation of σ, κ_i, and $\left(1-\frac{n_{vt}-1}{n_{vt}}\alpha_{vt}\right)$ is that (i) the measurement error in fertilizer or effort is uncorrelated with any of the independent variables, and (ii) there is no measurement error in the price of fertilizer. In this case, one can use OLS to estimate the following regression equation:

$$\log\left(\frac{f_{it}}{e_{it}}\right)=\varphi_i+\phi_{vt}-\sigma\log(p_{it})+\epsilon_{it},$$

where φ_i are household fixed effects and ϕ_{vt} are village-month fixed effects. This regression estimates κ_i and $\left(1-\frac{n_{vt}-1}{n_{vt}}\alpha_{vt}\right)$ by means of the household and village-month fixed effects, respectively. This estimation strategy relies on the assumption that risk-sharing is not household-specific; otherwise, φ_i would be also capturing variation in risk-sharing at the household level. Table 8 reports the results of estimating the previous regression. As one can see $\widehat{\sigma}\approx .6$. In order to back out the households' marginal disutility of effort, I take the estimated household fixed effects and divide them by $\widehat{\sigma}$. This gives $\widehat{\log(\kappa_i)}$. Finally, I use the fact that $\widehat{k}_i=\exp\left\{\widehat{\log(\kappa_i)}\right\}$. The average marginal disutility of effort is approximately 1.9. To get a sense of this number, assume that households have quadratic utility. Then, on average, the increase in consumption that would exactly compensate household i for an increase in one hour of work (i.e., the marginal rate of substitution of effort for consumption) is pinned down by the following equation:

$$\frac{\mathrm{d}c_i(\alpha)}{\mathrm{d}e_i} = \frac{1.9}{\rho c_i(\alpha)}.$$

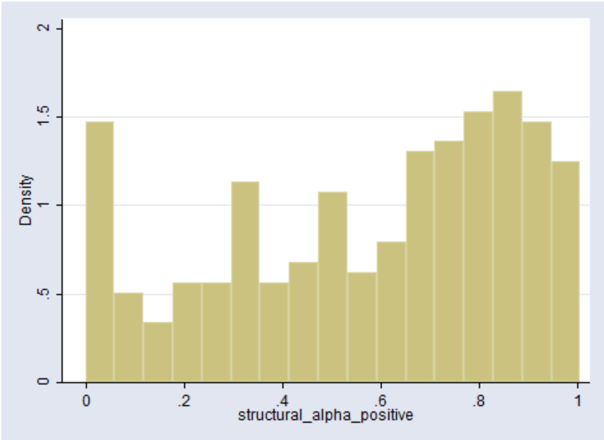
As average household consumption is approximately 117 rupees, the increase in consumption compensating the average household for an additional hour of work is $0.016\rho^{-1}$ rupees. According to the estimates provided by the Indian government ([Indian Labour Bureau, 2010]), in 2009, the daily wage rate for an adult male agricultural worker falls in the range of 50 to 120 2009 rupees, which roughly correspond to a hourly wage rate (assuming eight hours of work per day) of 0.5 to 1.2 1975 rupees. If the labor market were competitive, then the marginal rate of substitution of effort for consumption would be equal to the hourly wage rate. This would imply a coefficient of absolute risk aversion between 0.032 and 0.013. These numbers are comparable with the coefficients of absolute risk aversion for medium stakes estimated by [Binswanger, 1981], which measures ICRISAT farmers' risk attitudes by means of surveys and experiments.

One can also back out $\widehat{\zeta}_{vt} := \left(1 - \widehat{\frac{n_{vt}-1}{n_{vt}}\alpha_{vt}}\right)$ using the same procedure employed to obtain the households' disutility of effort. Clearly, n_{vt} and α_{vt} cannot be separately identified. However, following the standard practice in the literature ([Ligon et al., 2002], [Laczó, 2015], [Bold and Broer, 2016]), I can set village size equal to the number of households sampled by ICRISAT and back out a structural estimate of risk-sharing at the village-month level, $\widehat{\alpha_{vt}}$, by computing

$$\widehat{\alpha_{vt}} = \left(1 - \widehat{\zeta}_{vt}\right)\frac{\widetilde{n}_{vt}}{\widetilde{n}_{vt} - 1},$$

where $\widetilde{n}_{vt}$ is the imputed number of households sampled by ICRISAT. The number of households observed for each village and month is rather small: on average, less than 40 observations are used to compute a village-month fixed effect. This implies that $\widehat{\zeta}_{vt}$ is likely to be imprecisely estimated. By construction, $\zeta \in [0, 1)$; however, one sixth of the estimates of $\widehat{\alpha_{vt}}$ (101 out of 634) fall out of this range, being bigger than one. These observations cannot be used to estimate α_{vt}, because they would imply a negative α_{vt}, which does not make sense. If I drop the $\widetilde{n}_{vt}$ that are bigger than one, I obtain the α_{vt}'s plotted in Figure (1). On average, $\widehat{\alpha}_{vt}$ is equal to 0.6, but the estimates are more concentrated on the right of the distribution, with the

Figure 1: Kernel Density Estimate of $\widehat{\log(\kappa_{iv})}$



median being 0.7. These seem very reasonable numbers. I could also estimate a restricted OLS, in which I restrict $\left(1 - \widehat{\frac{n_{vt}-1}{n_{vt}}\alpha_{vt}}\right) \in (0,1)$ and $\widehat{\sigma} \in [0,1]$. I build the program and everything, but it seems that the algorithm is not converging...

Comparative Statics. I consider the following comparative statics exercise: how do choices of fertilizer and effort change when insurance changes? From Equation (13), once parameters σ, κ_i, and n_{vt} are pinned down, I can freely choose the sharing parameter, α, to analyze its effect on the expected relative choices of fertilizer over effort, $\mathbb{E}\left(\frac{f_{it}}{e_{it}}\right)$. Parameters σ and κ_i are pinned down by the estimates $\widehat{\sigma}$ and $\widehat{\kappa_i}$, obtained in the previous paragraph. As for n_{vt}, I follow the standard practice in the literature ([Ligon et al., 2002], [Laczó, 2015], [Bold and Broer, 2016]) and set village size equal to the number of households sampled by ICRISAT. Formally, I compute

$$\widetilde{\log\left(\frac{f_{it}}{e_{it}}\right)} = \widehat{\sigma}\widehat{\log(\kappa_i)} - \widehat{\sigma}\log\left(1 - \frac{\widetilde{n}_{vt}-1}{\widetilde{n}_{vt}}\widetilde{\alpha}\right) - \widehat{\sigma}\log(p_{ivt}),$$

where $\widetilde{n}_{vt}$ is the number of households sampled by ICRISAT, $\widetilde{\alpha}$ is imputed, and $\widetilde{\log\left(\frac{f_{it}}{e_{it}}\right)}$ is the resulting choice of fertilizer over effort. Figure 2 shows the kernel density estimate of $\widetilde{\log\left(\frac{f_{it}}{e_{it}}\right)}$ for $\alpha = 0$ (black) and $\alpha = 1$ (grey). The summary statistics of $\widetilde{\log\left(\frac{f_{it}}{e_{it}}\right)}$ for $\alpha = 0$ and $\alpha = 1$ are reported in Table 9. Hence, notice that, on average, when going from no insurance ($\alpha = 0$) to full insurance ($\alpha = 1$), the growth rate of fertilizer over effort is $2.1212 + .2517 = 2.3729$; i.e., fertilizer over effort is more than four times higher under full sharing than under autarky.

Figure 2: Counterfactural Policy Analysis

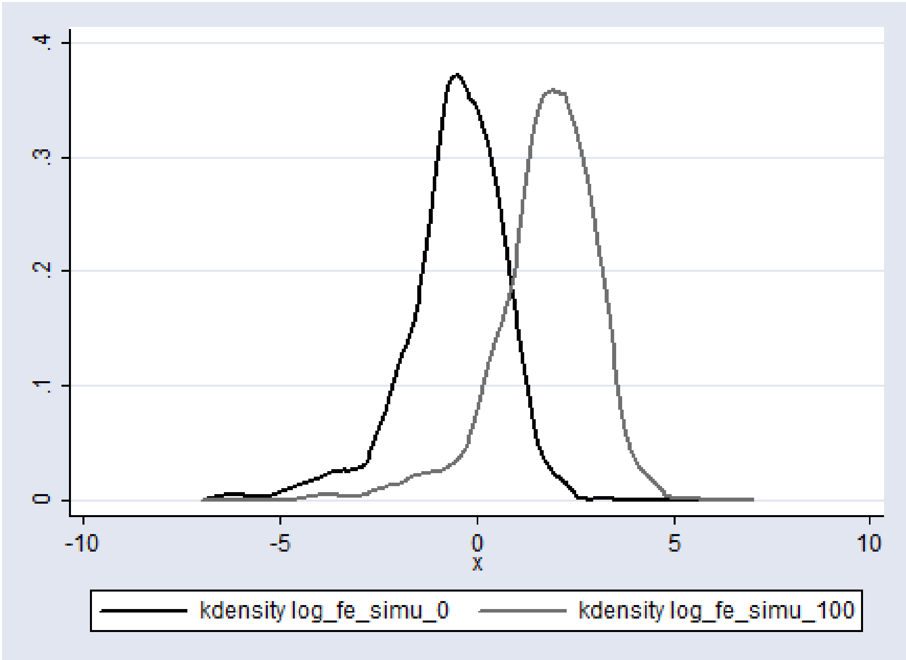


Table 9: Summary Statistics for $\widetilde{\log\left(\frac{f_{it}}{e_{it}}\right)}$.

	Average	S.d.	Min	Max
$\alpha = 0$	-.2517	1.1874	-6.5593	4.0445
$\alpha = 1$	2.1212	1.2159	-4.3519	6.5526

The intuition behind this result is that both effort supply and fertilizer use decrease when moving from autarky to full sharing; however, effort supply is more responsive to changes in risk-sharing than fertilizer use, and hence goes down more than what fertilizer use does. This simple calculation highlights that risk-sharing is a quantitatively important factor in shaping households' effort supply and fertilizer use.

The counterfactural policy analysis presented up to this showed the quantitative effect of risk-sharing on fertilizer per hours worked. Next, let me disentangle the effect of risk-sharing on effort supply and fertilizer use. Since land is fixed, household i's problem can be written as

$$\max_{e_i, f_i} \ell_i^{1-\chi} \left[e_i^{\frac{\sigma-1}{\sigma}} + f_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\chi\sigma}{\sigma-1}} - p f_i - \frac{\kappa}{1 - \frac{n-1}{n}\alpha} e_i.$$

This is equivalent to a profit maximization problem for a competitive firm. Since cost minimization is a necessary condition for profit maximization, consider the following cost minimization problem:

$$\min_{e_i,f_i} pf_i + \frac{\kappa}{1-\frac{n-1}{n}\alpha}e_i$$
$$\text{subject to } \ell_i^{1-\chi}\left[e_i^{\frac{\sigma-1}{\sigma}} + f_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\chi\sigma}{\sigma-1}} \geq \widehat{y}_i$$

Since land is fixed, this cost minimization problem is equivalent to

$$\min_{e_i,f_i} pf_i + \frac{\kappa}{1-\frac{n-1}{n}\alpha}e_i$$
$$\text{subject to } \left[e_i^{\frac{\sigma-1}{\sigma}} + f_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \geq y_i^{\dagger},$$

where $y_i^{\dagger} := \left(\frac{\widehat{y}_i}{\ell_i^{1-\chi}}\right)^{\frac{1}{\chi}}$. By the standard cost minimization problem with a CES technology, one gets

$$e_i^* = \frac{y_i^{\dagger}K^{-\sigma}}{p^{1-\sigma}+K^{1-\sigma}}$$

and

$$f_i^* = \frac{y_i^{\dagger}p^{-\sigma}}{p^{1-\sigma}+K^{1-\sigma}},$$

where $K := \frac{\kappa_i}{1-\frac{n-1}{n}\alpha}$. Taking logs, one gets

$$\log(e_i^*) = \log\left(y_i^{\dagger}\right) - \sigma\log(\kappa) + \sigma\log\left(1-\frac{n-1}{n}\alpha\right) - \log\left(p^{1-\sigma} + \left(\frac{\kappa}{1-\frac{n-1}{n}\alpha}\right)^{1-\sigma}\right)$$

and

$$\log(f_i^*) = \log\left(y_i^{\dagger}\right) - \sigma\log(p) - \log\left(p^{1-\sigma} + \left(\frac{\kappa}{1-\frac{n-1}{n}\alpha}\right)^{1-\sigma}\right).$$

Using the structural estimates, and setting village size equal to the number of households sampled by ICRISAT, one can simulate the choices of fertilizer and effort for different levels of α. The only issue is that $y_i^{\dagger}$ is unobserved. To avoid this problem, I consider the growth rates of effort and fertilizer when moving from α_0 to α_1. In econometric terms, these growth rates

Table 10: Summary Statistics of the Growth Rates of Effort and Fertilizer Use (from $\alpha = 0$ to $\alpha = 1$).

	Average	S.d.	Min	Max
$\log(e_{it}(0)) - \log(e_{it}(1))$	-3.260	.3312	-4.068	-2.246
$\log(f_{it}(0)) - \log(f_{it}(1))$	-.9520	.2646	-1.529	-.0387

are given by

$$\begin{aligned}\frac{e_{it}(\alpha_1) - e_{it}(\alpha_0)}{e_{it}(\alpha_0)} =& \log(e_{it}(\alpha_1)) - \log(e_{it}(\alpha_0)) \\ =& \widehat{\sigma}\log\left(\frac{\widetilde{n}_{vt}-1}{\widetilde{n}_{vt}}\alpha_1\right) - \log\left(\frac{\widetilde{n}_{vt}-1}{\widetilde{n}_{vt}}\alpha_1\right) + \\ & \log\left(p_{ivt}^{1-\widehat{\sigma}} + \left(\frac{\widehat{\kappa}_i}{1-\frac{\widetilde{n}_{vt}-1}{\widetilde{n}_{vt}}\alpha_0}\right)^{1-\widehat{\sigma}}\right) - \log\left(p_{ivt}^{1-\widehat{\sigma}} + \left(\frac{\widehat{\kappa}_i}{1-\frac{\widetilde{n}_{vt}-1}{\widetilde{n}_{vt}}\alpha_1}\right)^{1-\widehat{\sigma}}\right),\end{aligned}$$

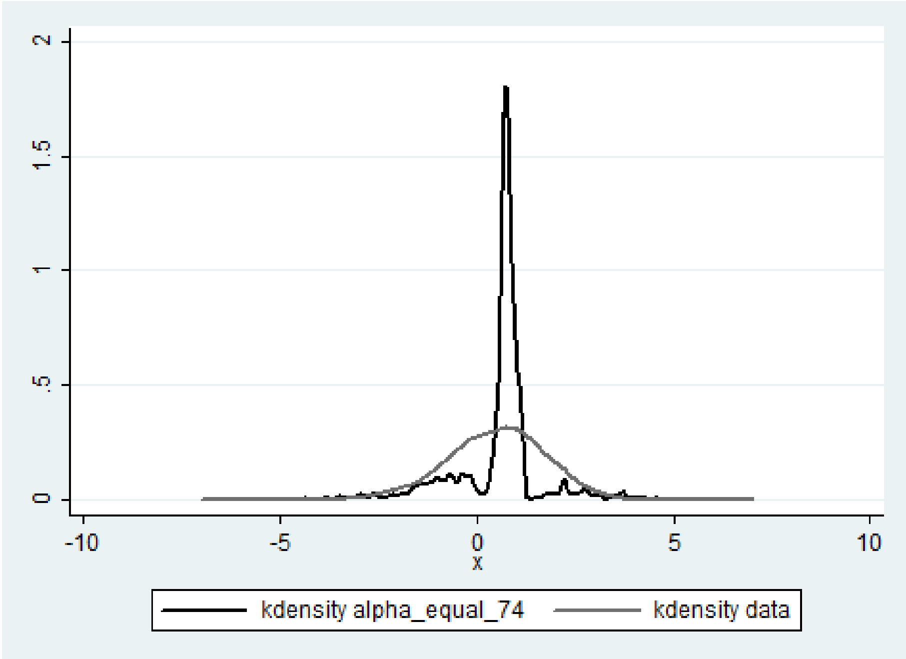
and

$$\begin{aligned}\frac{f_{it}(\alpha_1) - f_{it}(\alpha_0)}{f_{it}(\alpha_0)} =& \log(f_{it}(\alpha_1)) - \log(f_{it}(\alpha_0)) \\ =& \log\left(p_{ivt}^{1-\widehat{\sigma}} + \left(\frac{\widehat{\kappa}_i}{1-\frac{\widetilde{n}_{vt}-1}{\widetilde{n}_{vt}}\alpha_0}\right)^{1-\widehat{\sigma}}\right) - \log\left(p_{ivt}^{1-\widehat{\sigma}} + \left(\frac{\widehat{\kappa}_i}{1-\frac{\widetilde{n}_{vt}-1}{\widetilde{n}_{vt}}\alpha_1}\right)^{1-\widehat{\sigma}}\right).\end{aligned}$$

Notice that these growth rates are independent of $y_i^{\dagger}$. Tables report the summary statistics of the growth rates of effort and fertilizer use when going from $\alpha = 0$ to $\alpha = 1$ are reported in Table 10. When going from no insurance to full insurance is, the average growth rate of fertilizer use is -95%; hence, on average, fertilizer use is halved. On the other hand, on average, effort supply decreases by more than 6 times. Which households are more affected by insurance?

Validating the Structural Model of Exogenous Risk-Sharing. The reduced-form test for perfect risk-sharing conducted in Subsection 3.2 suggests that $\alpha \approx 0.74$ (see Table 2). Given this, I can consider the distribution of simulated choices of fertilizer over effort, $\widetilde{\log\left(\frac{f_{it}}{e_{it}}\right)}$, for $\alpha = 0.74$ and compare it with the actual distribution of fertilizer over effort, $\log\left(\frac{f_{it}}{e_{it}}\right)$. Figure 3 does exactly that. The grey line is the density of $\widetilde{\log\left(\frac{f_{it}}{e_{it}}\right)}$ when $\alpha = 0.74$, while the grey line is the density of the data. As one can see, while the median and the mean of the simulated distribution are close to the the median and the mean of the actual distribution of fertilizer

Figure 3: Kernel Density Estimate of $\log\widetilde{\left(\frac{f_{ivt}}{e_{ivt}}\right)}$ vs $\log\left(\frac{f_{ivt}}{e_{ivt}}\right)$.



use per hours worked, the dispersion in the data is much larger. This suggests that there are relevant sources of variation in households' choices of fertilizer over effort that I do not take into account in the simple model of exogenous risk-sharing.

Next, I compute a sharing parameter α that makes the mean of $\log\left(\frac{f_{it}}{e_{it}}\right)$ match the mean of fertilizer use over hours worked. To do so, I pick an α to minimizes the mean squared difference between $\log\left(\frac{f_{it}}{e_{it}}\right)$ and $\log\widetilde{\left(\frac{f_{it}}{e_{it}}\right)}$. In this case, $\alpha = 0.82$, which is a very reasonable number. I also compute an α that makes the median of the simulated distribution match the median of the actual distribution. I do so by choosing an α to minimizes the mean absolute difference between $\log\left(\frac{f_{it}}{e_{it}}\right)$ and $\log\widetilde{\left(\frac{f_{it}}{e_{it}}\right)}$. In this case, $\alpha = 0.75$, which is extremely close to sharing parameter suggested by the reduced-form test for perfect risk-sharing.

Optimal α!

Endogenous Risk-Sharing with Linear Contracts: The Effect of a Fertilizer Subsidy. because it allows me to study the effect of different policies on the level of village insurance and social welfare. A policy that naturally comes to mind are fertilizer subsidies. These policies are extremely important in India.[17]

Consider the following production function:

$$y\left(\boldsymbol{a}_i\right)=\ell_i^{1-\chi}\left[e_i^{\frac{\sigma-1}{\sigma}}+f_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma\chi}{\sigma-1}}.$$

Let $P:=\left(K^{1-\sigma}+p^{1-\sigma}\right)^{\frac{1}{q-\sigma}}$. The demands for fertilizer and effort are given by

$$f_i^*=\left(\frac{p}{P}\right)^{-\sigma}\left(\frac{\chi}{P}\right)^{\frac{1}{1-\chi}}\ell_i$$

and

$$e_i^*=\left(\frac{K}{P}\right)^{-\sigma}\left(\frac{\chi}{P}\right)^{\frac{1}{1-\chi}}\ell_i$$

From this equation, I can compute $\frac{\partial e_i^*}{\partial\alpha}$. First, I make use of the chain rule to write

$$\frac{\partial e_i^*}{\partial\alpha}=\frac{\partial e_i^*}{\partial K}\frac{\partial K}{\partial\alpha}.$$

Then, notice that

$$\frac{\partial K}{\partial\alpha}=\frac{\kappa}{\left(1-\frac{n-1}{n}\alpha\right)^2}\left(\frac{n-1}{n}\right)$$

and

$$\frac{\partial e_i^*}{\partial K}=\chi^{\frac{1}{1-\chi}}\ell_i\left[-\sigma K^{-\sigma-1}P^{\sigma-\frac{1}{1-\chi}}+K^{-2\sigma}\left(\sigma-\frac{1}{1-\chi}\right)P^{\sigma-\frac{1}{1-\chi}-1}\left(K^{1-\sigma}+p^{1-\sigma}\right)^{\frac{1}{1-\sigma}-1}\right].$$

Finally, notice that

$$\frac{\partial\mathbb{V}\mathrm{ar}\left(c_i\left(\alpha\right)\right)}{\partial\alpha}=\Sigma^2\left(-2\left(1-\alpha\right)+2\frac{\alpha}{n}+\frac{\left(1-2\alpha\right)}{n}\right).$$

Let α^* be an optimal sharing rule under private information. By Claim 4, if $\alpha^*\in(0,1)$, then

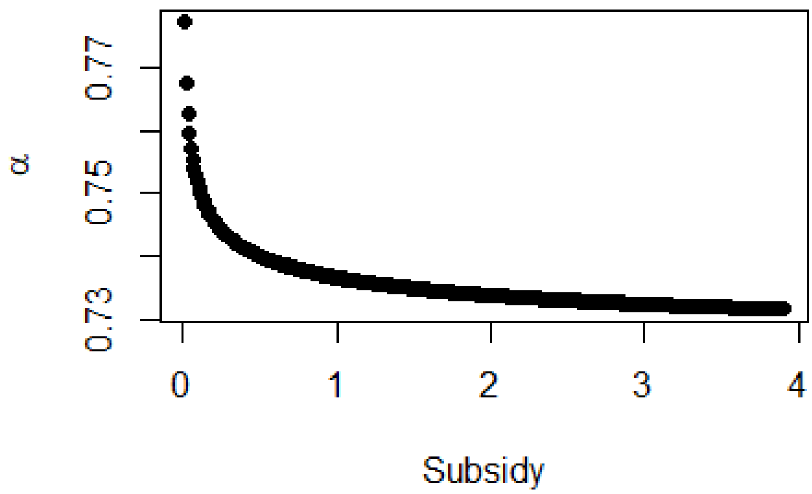
[17]There is also a Ministry of Chemicals and Fertilizers in India: http://fert.nic.in/page/fertilizer-policy.

it must be the case that

$$\underbrace{\sum_{i\in N}\left(\kappa\left(\frac{1}{1-\frac{n-1}{n}\alpha}-1\right)\frac{\partial e_i^*}{\partial\alpha}\right)}_{(-)}\underbrace{-\frac{n\rho}{2}\frac{\partial\mathbb{V}\text{ar}\left(c_i\left(\alpha\right)\right)}{\partial\alpha}}_{(+)}=0. \tag{14}$$

My aim is to solve Equation (14). To do so, I need values for χ and ρ. To obtain values for χ and ρ, I do the following. I build a grid of possible values for χ and ρ ($\chi \in (0,1)$ and $\rho \in (0,2.5]$). Then, I solve Equation (14) to get an optimal α. Then, I pick χ and ρ to minimize either the absolute distance or the Euclidean distance between the solution to Equation (14) and $\alpha = 0.75$, which is what I obtained to match the average of simulated choices of fertilizer per hours worked with actual choices of fertilizer per hours worked. If I do that, I obtain $(\chi,\rho) = (0.42, 1.6)$ or $(\chi,\rho) = (0.43, 2.1)$. Figure 4 shows the optimal sharing as a function of a fertilizer subsidy τ such that $p' = \tau p$, for $\tau \in (0,4)$. When moving from normal price

Figure 4: Optimal Sharing vs. Fertilizer Subsidy



to *price* $\times$ 0.1, risk-sharing goes from 0.75 to 0.78; i.e., it increases by 4%. Notice that the higher is the subsidy, the higher is risk-sharing. The intuition is the following. When the price of fertilizer is subsidized, people use more fertilizer, and since effort and fertilizer are complements, they also want to use more effort. Hence, you move to a flatter region of the production function. Notice that the benefit of insurance, in terms of reduction of the variance of consumption, is independent of the price. However, the cost of insurance is affected by the price of fertilizer, because fertilizer price affects effort provision, and this affects the impact of insurance on effort provision. Since you are in a flatter region, impact of insurance on effort provision is smaller. Hence, cost of insurance is lower. Hence, you want to increase α, because marginal cost becomes smaller than marginal benefit.

By the way, another interesting implication of my model is that if you don't consider the countervailing effect of insurance on fertilizer use, you would underestimate the elasticity of fertilizer to subsidy. What agricultural economists normally do when they want to estimate production functions, input demands, etc., is to specify a translog production function, which can be conceived as a linear approximation to a CES production function. Then, they use Sheppard lemma to say that

$$\frac{\partial \pi_{it}}{\partial p_{it}^{f}} = f_{it}^{*} = \beta_f + \beta_{fy} \log(y_{it}) + \beta_{fe} \log(p_{it}^{e}) + \beta_{ff} \left(p_{it}^{f}\right).$$

Suppose that you don't think that insurance matters. Then, you would estimate

$$\frac{\partial f_{it}^{*}}{\partial p_{it}^{f}} = \beta_{ff} \frac{1}{p_{it}^{f}}.$$

But in reality, because of the model of endogenous risk-sharing blablabla, we know that

$$\frac{\partial f_{it}^{*}}{\partial p_{it}^{f}} = \beta_{fe} \frac{1}{p_{it}^{e}} \frac{\partial p_{it}^{e}}{\partial \alpha} \frac{\partial \alpha}{\partial p_{it}^{f}} + \beta_{ff} \frac{1}{p_{it}^{f}}.$$

Since effort and fertilizer are complements, $\beta_{fe} < 0$; moreover, we have shown above that $\frac{\partial \alpha}{\partial p_{it}^{f}}$ is negative. So, in reality, noticing that $\beta_{ff} < 0$ and that also $\frac{\partial f_{it}^{*}}{\partial p_{it}^{f}} < 0$,

$$\frac{\partial f_{it}^{*}}{\partial p_{it}^{f}} > \beta_{ff} \frac{1}{p_{it}^{f}},$$

or more intuitively,

$$\left| \frac{\partial f_{it}^{*}}{\partial p_{it}^{f}} \right| < \left| \beta_{ff} \frac{1}{p_{it}^{f}} \right|$$

This is intuitive: if you decrease the price of fertilizer demand for fertilizer will increase yes, but, since insurance also increases, fertilizer demand will go down a little bit (there's a countervailing force). So, in absolute terms, the elasticity of fertilizer to a subsidy is lower than what you would expect!

Notice that the objective function is written

$$y\left(e_i^*, f_i^*\right) - pf_i^* - ke_i^* - \frac{\rho}{2}\left(\left(1-\alpha\right)^2 + \frac{\alpha^2}{n} + \frac{\alpha\left(1-\alpha\right)}{n}\right)\sigma^2$$

Obtaining a closed-form solution for α^* is unfeasible even in this relatively simple model. Hence, I proceed as follows. Consider the case in which fertilizer is the only agricultural input; i.e., $\boldsymbol{z}_i = f_i$. First, I take the price of fertilizer, p, as a given. I use the IC constraints to find the input demands for effort and fertilizer for a generic α, $e_i^*\left(\alpha\right)$ and $f_i^*\left(\alpha\right)$. Then, I substitute $e_i^*\left(\alpha\right)$ and $f_i^*\left(\alpha\right)$ in the planner's objective function, and maximize with respect to α. This gives me the optimal sharing rule as a function of p. Then, I imagine that the government sets a fertilizer subsidy, which gives rise to a new price of fertilizer, p'. Hence, I repeat the procedure described above for a price of fertilizer equal to p'.

Optimal fertilizer subsidy. Suppose the government wants to maximize social welfare. He ought to maximize

$$y\left(e_i^*, f_i^*\right) - pf_i^* - ke_i^* - \frac{\rho}{2}\mathbb{V}\text{ar}\left(c_i\left(\alpha^*\right)\right).$$

Doing the FOC wrt to p, one gets

$$\frac{\partial y\left(e_i^*, f_i^*\right)}{\partial e_i^*}\frac{\partial e_i^*}{\partial p}$$

Wait a second. https://www.jstor.org/stable/1349278?seq=1#page_scan_tab_contents. It seems a well-known result that, by the envelope theorem, effect of an input price change on profit is equal the negative of the quantity of that input. If you can show that the planner's problem is equivalent to maximizing aggregate profits minus $\rho/2$ times variance... then you can get a closed form solution. So,

$$\frac{\partial Varc_i(\alpha)}{\partial \alpha} = 2\alpha - \frac{2n-1}{n}...$$

Then derivative of social welfare wrt alpha would be something like

$$-\sum_i e_i^* - \frac{\rho}{2} 2\alpha - \frac{2n-1}{n}?$$

The aim is to understand how a change in fertilizer price affects optimal risk-sharing. Profits are zero for a competitive firm operating a constant returns to scale technology.[18] By the standard cost minimization problem with a CES technology, one gets

$$e_{ivt}^* = \frac{\widehat{y}_{ivt} K_{iv}^{-\sigma}}{p_{ivt}^{1-\sigma} + K_{vt}^{1-\sigma}}$$

and

$$f_{ivt}^* = \frac{\widehat{y}_{ivt} p_{ivt}^{-\sigma}}{p_{ivt}^{1-\sigma} + K_{iv}^{1-\sigma}},$$

where $K_{vt} := \frac{k_{iv}}{1-\frac{n_{vt}-1}{n_{vt}}\alpha}$ (check `http://www.yorku.ca/bucovets/5010/consumer/s04.pdf`). Hence,

$$\mathbb{E}\left(c_{ivt}(\alpha)\right) = (1-\alpha)\left(y\left(e_{ivt}^*, f_{ivt}^*\right) - p \cdot f_{ivt}^*\right) + \alpha \frac{\sum_{j \in N} y\left(e_{jvt}^*, f_{jvt}^*\right) - p \cdot f_{jvt}^*}{n} - \kappa e_{ivt}^*$$

and

$$\mathbb{V}\mathrm{ar}\left(c_{ivt}(\alpha)\right) = \left((1-\alpha)^2 + \frac{\alpha^2}{n} + \frac{\alpha(1-\alpha)}{n}\right)\sigma^2.$$

Production at the optimum is

$$y\left(\boldsymbol{a}_{ivt}^*\right) = \left[e_{ivt}^{*\frac{\sigma-1}{\sigma}} + f_{ivt}^{*\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

[18]To see this, recall that if the production function is homogeneous of degree one (i.e., exhibits constant returns to scale) then the cost function is homogeneous of degree one in y. By Euler's homogeneous function theorem, $c'(q)\,q = c(q)$. Firm's revenue is $pq - c(q)$. I.e., $pq - c'(q)\,q$. If $p = c'(q)$ then profits are 0 for any q. Notice that $p = c'(q)$ at an optimum: FOCs are $p = c'(q)$.

Hence, substituting $e_i^*(\alpha)$ and $z_i^*(\alpha)$ into the production function, I get

$$y(\boldsymbol{a}_i^*) = \frac{\widehat{y}_{ivt}}{p_{ivt}^{1-\sigma} + K_{vt}^{1-\sigma}} \left[K_{iv}^{1-\sigma} + p_{ivt}^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}} = \widehat{y}_{ivt} \left[K_{iv}^{1-\sigma} + p_{ivt}^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}-1}.$$

And then you substitute this into the objective function and pick alpha to minimize it. I.e.,

$$\begin{aligned}
\text{Obj}_i(\alpha) = (1-\alpha) & \left(\widehat{y}_{ivt} \left[K_{iv}^{1-\sigma} + p_{ivt}^{1-\sigma}\right]^{\frac{1}{\sigma-1}} - p_{ivt} \cdot \frac{\widehat{y}_{ivt} p_{ivt}^{-\sigma}}{p_{ivt}^{1-\sigma} + K_{iv}^{1-\sigma}}\right) \\
& + \alpha \frac{\sum_{j \in N} \widehat{y}_{jvt} \left[K_{jv}^{1-\sigma} + p_{jvt}^{1-\sigma}\right]^{\frac{1}{\sigma-1}} - p_{jvt} \cdot \frac{\widehat{y}_{jvt} p_{jvt}^{-\sigma}}{p_{jvt}^{1-\sigma} + K_{iv}^{1-\sigma}}}{n} - \kappa_{iv} \frac{\widehat{y}_{ivt} K_{iv}^{-\sigma}}{p_{ivt}^{1-\sigma} + K_{vt}^{1-\sigma}} \\
& - \frac{\rho}{2} \left((1-\alpha)^2 + \frac{\alpha^2}{n} + \frac{\alpha(1-\alpha)}{n}\right) \sigma^2
\end{aligned}$$

and

$$\text{Obj}(\alpha) = \sum_i \text{Obj}_i(\alpha).$$

Estimating the Income Process. Before taking the model of endogenous α to the data, it is necessary to obtain an estimate of the variance of the income shock, σ^2. The model is very simple, as it assumes that households' choices are made before the realization of the shock. Hence, given that I observe price of fertilizer, fertilizer use, and effort, one very simple way of doing the estimation is to write

$$\widehat{\varepsilon}_{ivt} = \pi_{ivt} - y(e_{ivt}, f_{ivt}) - pf_{ivt},$$

and then simply calculate

$$\mathbb{V}\text{ar}(\widehat{\varepsilon}_{ivt}).$$

Clearly, in reality, it might be the case that households' choices are correlated to the shock, as they are not made ex ante. Then, one way through this would be to get instruments for effort and fertilizer.

4 Conclusions/Further Research

While rural households in low-income countries face sizable income risks, they often lack access to formal insurance. Despite this shortfall, these households do manage to smooth their consumption, albeit imperfectly, by relying on informal insurance arrangements. Studies on risk-sharing abound, but few of them relate informal insurance to other facets of village economies. This paper analyzes the link between informal insurance and agricultural input use, an extremely relevant topic in today's academic and policy circles. On the one hand, informal insurance may well have a discouraging effect on households' incentives to exert effort, as economists have long been arguing. On the other, effort exhibits different degrees of complementarity and substitutability with distinct agricultural intermediates, such as fertilizer. For this reason, informal insurance may impact households' choices of agricultural inputs in village economies. This paper shows theoretically that better insured households decrease effort provision, increase the use of inputs substituting effort, and decrease the use of inputs complementing it. The paper then proceeds by using the last ICRISAT panel from rural India to provide evidence that (i) insurance and effort supply are negatively correlated, (ii) effort supply and fertilizer use are positively correlated, and (iii) more insurance is associated to reductions in fertilizer use. Finally, the paper structurally estimates a simple model of exogenous risk-sharing to quantify the importance of risk-sharing on fertilizer use and effort supply.

While the results of the structural estimation suggests an important role for risk-sharing in shaping household choices of effort and fertilizer use, one can immediately see that they suffer from some obvious concerns. First, and most importantly, risk-sharing is likely to be an endogenous object. To tackle this issue, my aim is to structurally estimate a full-fledged model of optimal risk-sharing. The estimation of such a model is interesting because it allows one to analyze the effect of changes in fertilizer prices on informal insurance and social welfare. For example, I could consider the effect of the introduction of a fertilizer subsidy on risk-sharing and welfare. To be sure, this policy would undoubtedly increase fertilizer use. As effort and fertilizer are complements, a fertilizer subsidy would also increase effort supply for any given level of risk-sharing, making it easier for the planner to satisfy the households' incentive-compatibility constraints. Hence, my conjecture is that in a model of endogenous risk-sharing a fertilizer subsidy would increase both fertilizer use and risk-sharing, shifting the economy closer to the full information benchmark. It would be interesting to understand the welfare gains of this policy, and to analyze how these gains change when the endogenous response of risk-sharing is considered. This can be done by contrasting the welfare effects of the same policy in a model of exogenous risk-sharing and in a model of endogenous risk-sharing.

A Proofs

Proof of Claim 1. The planner's problem under full information is equivalent to

$$\max_{\boldsymbol{e},\boldsymbol{z}} \sum_{i\in N} \lambda_i \left((1-\alpha)\left(y\left(e_i, \boldsymbol{z}_i\right) - \boldsymbol{p}\cdot\boldsymbol{z}_i\right) + \alpha \frac{\sum_{j\in N} y\left(e_j, \boldsymbol{z}_j\right) - \boldsymbol{p}\cdot\boldsymbol{z}_j}{n} - \kappa e_i \right).$$

If $(e^*, \boldsymbol{z}^*)$ is an interior solution to problem (4), then

$$\lambda_i \left((1-\alpha)\, y_e\left(e_i^*, \boldsymbol{z}_i^*\right) + \alpha \frac{y_e\left(e_i^*, \boldsymbol{z}_i^*\right)}{n} - k \right) + \sum_{j\neq i} \lambda_i \left((1-\alpha)\, \frac{y_e\left(e_i^*, \boldsymbol{z}_i^*\right)}{n} - k \right) = 0,$$

for each $i \in N$. Using the fact that $\sum_{j\in N} \lambda_j = 1$,

$$y_e\left(e_i^*, \boldsymbol{z}_i^*\right) = k;$$

i.e., the marginal cost of effort equals its marginal product. The same argument holds for the agricultural intermediates. □

Proof of Claim 2. The planner's problem under private information is equivalent to

$$\max_{\boldsymbol{e}_i,\boldsymbol{z}_i} \left(1 - \frac{n-1}{n}\alpha\right) y\left(\left(e_i, \boldsymbol{z}_i\right) - \boldsymbol{p}\cdot\boldsymbol{z}_i\right) - ke_i,\ \forall i \in N.$$

If $(e^*, \boldsymbol{z}^*)$ is an interior solution to problem (4), then

$$\left(1 - \frac{n-1}{n}\alpha\right) y_e\left(e_i^*, \boldsymbol{z}_i^*\right) - k = 0$$

and

$$\left(1 - \frac{n-1}{n}\alpha\right) \left(y_{\boldsymbol{z}}\left(e_i^*, \boldsymbol{z}_i^*\right) - \boldsymbol{p}\right) = 0,$$

for each $i \in N$. □

Proof of Theorem 1. Notice that p_e is decreasing in α. By the law of supply, the demand for an input is decreasing in its price. Hence, e_i^* is decreasing in α.

Moreover, α only affects p_e. Hence,

$$\begin{aligned}\frac{\partial z_i^{q*}(p_e,\boldsymbol{p})}{\partial \alpha} &= \frac{\partial z_i^{q*}(p_e,\boldsymbol{p})}{\partial p_e}\frac{\partial p_e}{\partial \alpha} \\ &= \frac{\partial z_i^{q*}(p_e,\boldsymbol{p})}{\partial p_e}\left(-\left(1-\frac{n-1}{n}\alpha\right)^{-1}\left(-\frac{n-1}{n}\right)\right).\end{aligned}$$

□

Proof of Claim 3. The problem of finding an optimal linear contract when there is full information is

$$\max_{\alpha\in[0,1]}\sum_{i\in N}\left(\mathbb{E}\left(c_i(\alpha)\right)-\frac{\rho}{2}\mathbb{V}\mathrm{ar}\left(c_i(\alpha)\right)-\kappa e_i\right),$$

where

$$\mathbb{E}\left(c_i(\alpha)\right)=(1-\alpha)\left(y\left(e_i,\boldsymbol{z}_i\right)-\boldsymbol{p}\cdot\boldsymbol{z}_i+\mu\right)+\alpha\frac{\sum_{j\in N}y\left(e_j,\boldsymbol{z}_j\right)-\boldsymbol{p}\cdot\boldsymbol{z}_j+\mu}{n}-\kappa e_i$$

and

$$\mathbb{V}\mathrm{ar}\left(c_i(\alpha)\right)=\left((1-\alpha)^2+\frac{\alpha^2}{n}+\frac{\alpha(1-\alpha)}{n}\right)\Sigma^2.$$

Notice that

$$\sum_{i\in N}\mathbb{E}\left(c_i(\alpha)\right)=\sum_{i\in N}\left(y\left(e_i,\boldsymbol{z}_i\right)-\boldsymbol{p}\cdot\boldsymbol{z}_i+\mu\right).$$

Recall that under full information, the choices of e_i and $\boldsymbol{z}_i$ are independent of α (see Claim 1). Hence, $\sum_i \mathbb{E}\left(c_i(\alpha)\right)$ does not depend on α. Moreover, it is easy to check that $\mathbb{V}\mathrm{ar}\left(c_i(\alpha)\right)$ is minimized when $\alpha=1$. Hence, under full information, $\alpha=1$. □

Proof of Claim 4. The problem of finding an optimal linear contract when there is private information is

$$\max_{\alpha\in[0,1]}\sum_{i\in N}\left(\mathbb{E}\left(c_i(\alpha)\right)-\frac{\rho}{2}\mathbb{V}\mathrm{ar}\left(c_i(\alpha)\right)-\kappa e_i\right)$$

subject to

$$\begin{aligned}&\left(1-\frac{n-1}{n}\alpha\right)y_e\left(e_i,\boldsymbol{z}_i\right)=\kappa,\\ &y_{\boldsymbol{z}}\left(e_i,\boldsymbol{z}_i\right)=\boldsymbol{p},\end{aligned}$$

Notice that

$$\sum_{i\in N}\mathbb{E}\left(c_i\left(\alpha\right)\right)=\sum_{i\in N}\left(y\left(e_i,\boldsymbol{z}_i\right)-\boldsymbol{p}\cdot\boldsymbol{z}_i+\mu\right).$$

Hence, the problem can be written as

$$\max_{\alpha\in[0,1]}\sum_{i\in N}\left(y\left(e_i^*,\boldsymbol{z}_i^*\right)-\boldsymbol{p}\cdot\boldsymbol{z}_i^*+\mu-\kappa e_i\right)-\frac{n\rho}{2}\mathbb{V}\mathrm{ar}\left(c_i\left(\alpha\right)\right)$$

Derivate the planner's objective function with with respect to α to obtain

$$\sum_{i\in N}\left(\frac{\partial y\left(e_i^*,\boldsymbol{z}_i^*\right)}{\partial e_i}\frac{\partial e_i^*}{\partial\alpha}+\frac{\partial y\left(e_i^*,\boldsymbol{z}_i^*\right)}{\partial z_i}\frac{\partial z_i^*}{\partial\alpha}-\boldsymbol{p}\cdot\frac{\partial\boldsymbol{z}_i^*}{\partial\alpha}-\kappa\frac{\partial e_i^*}{\partial\alpha}\right)-\frac{n\rho}{2}\frac{\partial\mathbb{V}\mathrm{ar}\left(c_i\left(\alpha\right)\right)}{\partial\alpha}$$

Rearranging, I get

$$\sum_{i\in N}\left(\left(\frac{\partial y\left(e_i^*,\boldsymbol{z}_i^*\right)}{\partial e_i}-\kappa\right)\frac{\partial e_i^*}{\partial\alpha}+\left(\frac{\partial y\left(e_i^*,\boldsymbol{z}_i^*\right)}{\partial z_i}-\boldsymbol{p}\right)\frac{\partial\boldsymbol{z}_i^*}{\partial\alpha}\right)-\frac{n\rho}{2}\frac{\partial\mathbb{V}\mathrm{ar}\left(c_i\left(\alpha\right)\right)}{\partial\alpha}.$$

From the IC constraints given in Claim 2, the previous expression boils down to

$$\sum_{i\in N}\left(\kappa\left(\frac{1}{1-\frac{n-1}{n}\alpha}-1\right)\frac{\partial e_i^*}{\partial\alpha}\right)-\frac{n\rho}{2}\frac{\partial\mathbb{V}\mathrm{ar}\left(c_i\left(\alpha\right)\right)}{\partial\alpha}.$$

Notice that $\left(\frac{1}{1-\frac{n-1}{n}\alpha}-1\right)>0$ because $\frac{n-1}{n}\alpha\in(0,1)$, $\frac{\partial e_i^*}{\partial\alpha}<0$ by the law of supply, and $\frac{\partial\mathbb{V}\mathrm{ar}(c_i(\alpha))}{\partial\alpha}<0$ (see Claim 3). □

Proof of Proposition 1. Let $\left(\boldsymbol{c}^*\left(\boldsymbol{\pi}\right),\boldsymbol{a}^*\right)$ be a solution to the planner's problem, and $\mu\left(\boldsymbol{\pi}\right)$ be the Lagrange multiplier of the feasibility constraint when profit profile $\boldsymbol{\pi}$ realizes. The first-order conditions for $c_i\left(\boldsymbol{\pi}\right)$ are

$$\lambda_i u'\left(c_i^*\left(\boldsymbol{\pi}\right)\right)\phi^{\boldsymbol{\pi}}\left(\boldsymbol{\pi}\mid\boldsymbol{a}^*\right)=\mu\left(\boldsymbol{\pi}\right).$$

Combining this with the first-order conditions for $c_j\left(\boldsymbol{\pi}\right)$ yields

$$\frac{u'\left(c_i^*\left(\boldsymbol{\pi}\right)\right)}{u'\left(c_j^*\left(\boldsymbol{\pi}\right)\right)}=\frac{\lambda_j}{\lambda_i},$$

for each i, j and each $\boldsymbol{\pi}$. That is, for each realization of profits, consumption is adjusted so that the ratio of marginal utilities between households is constant. $\square$

Proof of Claim 5. The first-order condition for e_i reads[19]

$$\frac{1}{\lambda_i} \sum_{j \in N} \lambda_j \int u\left(c_j^*(\boldsymbol{\pi})\right) \phi_{e_i}^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right) \prod_{k \neq i} \phi^{\pi_k}\left(\pi_k \mid \boldsymbol{a}_k^*\right) \mathrm{d}\boldsymbol{\pi} = k,$$

Notice that the right-hand side of this equation is i's private marginal cost of effort, while the right-hand size is the marginal increase in a weighted sum of the households' expected utility of consumption associated to a unitary increase in i's effort. On the other hand, the first-order condition for z_i^q is given by

$$\sum_{j \in N} \lambda_j \int u\left(c_i^*(\boldsymbol{\pi})\right) \phi_{z_i^q}^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right) \prod_{k \neq i} \phi^{\pi_k}\left(\pi_k \mid \boldsymbol{a}_k^*\right) \mathrm{d}\boldsymbol{\pi} = 0, \tag{15}$$

Recall that $\phi^{\pi^i}\left(\pi_i \mid \boldsymbol{a}_i\right) = \phi^{\varepsilon^i}\left(\pi_i - y\left(\boldsymbol{a}_i\right) + \boldsymbol{p}\boldsymbol{z}_i\right)$. Hence,

$$\phi_{z_i^q}^{\pi^i}\left(\pi_i \mid \boldsymbol{a}_i\right) = \phi_{\varepsilon_i}^{\varepsilon^i}\left(\pi_i - y\left(\boldsymbol{a}_i\right) + \boldsymbol{p}\boldsymbol{z}_i\right)\left[-y_{z^q}\left(\boldsymbol{a}_i\right) + p^q\right].$$

Thus, Equation (15) can be rewritten as

$$\left[-y_{z^q}\left(\boldsymbol{a}_i^*\right) + p^q\right] \sum_{j \in N} \lambda_j \int u\left(c_j^*(\boldsymbol{\pi})\right) \phi_{\varepsilon_i}^{\varepsilon^i}\left(\pi_i - y\left(\boldsymbol{a}_i^*\right) + \boldsymbol{p}\boldsymbol{z}_i^*\right) \prod_{k \neq i} \phi^{\pi_k}\left(\pi_k \mid \boldsymbol{a}_k^*\right) \mathrm{d}\boldsymbol{\pi} = 0,$$

which is true if and only if $y_{z^q}\left(\boldsymbol{a}_i^*\right) - p^q$. That is, the marginal product of intermediate q is equal to its price. $\square$

Proof of Proposition 2. Recall that $\phi^{\pi^i}\left(\pi_i \mid \boldsymbol{a}_i\right) = \phi^{\varepsilon^i}\left(\pi_i - y\left(\boldsymbol{a}_i\right) + \boldsymbol{p}\boldsymbol{z}_i\right)$. Hence,

$$\phi_{z_i^q}^{\pi^i}\left(\pi_i \mid \boldsymbol{a}_i\right) = \phi_{\varepsilon_i}^{\varepsilon^i}\left(\pi_i - y\left(\boldsymbol{a}_i\right) + \boldsymbol{p}\boldsymbol{z}_i\right)\left[-y_{z^q}\left(\boldsymbol{a}_i\right) + p^q\right].$$

[19]In the following, $\frac{\partial \phi^{\pi_i}(\pi_i \mid \boldsymbol{a}_i)}{\partial x} := \phi_x^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i\right)$.

As a consequence, the first-order condition for z_i^q can be rewritten as

$$\left[-y_{z^q}\left(\boldsymbol{a}_i\right)+p^q\right] \int u\left(c_i\left(\boldsymbol{\pi}\right)\right) \phi_{\varepsilon_i}^{\varepsilon^i}\left(\pi_i-y\left(\boldsymbol{a}_i\right)+\boldsymbol{p}\boldsymbol{z}_i\right) \prod_{j \neq i} \phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j\right) \mathrm{d} \boldsymbol{\pi}=0,$$

which is true as long as

$$y_{z^q}\left(\boldsymbol{a}_i\right)=p^q.$$

Hence, Problem (26) can be rewritten as

$$\begin{aligned}
& \max_{\boldsymbol{c}(\boldsymbol{\pi}), \boldsymbol{a}} \sum_{i \in N} \lambda_i\left\{\mathbb{E}_{\boldsymbol{\pi}}\left[u\left(c_i\left(\boldsymbol{\pi}\right)\right) \mid \boldsymbol{a}\right]-k e_i\right\} \\
& \text{subject to } \sum_i c_i\left(\boldsymbol{\pi}\right)=\sum_i \pi_i, \ \forall \boldsymbol{\pi}, \\
& \int u\left(c_i\left(\boldsymbol{\pi}\right)\right) \phi_{e_i}^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i\right) \prod_{j \neq i} \phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j\right) \mathrm{d} \boldsymbol{\pi}=k, \forall i, \\
& y_{z^q}\left(\boldsymbol{a}_i\right)=p^q, \ \forall i, \forall q .
\end{aligned} \tag{16}$$

The Lagrangian associated to Problem (16) is

$$\begin{aligned}
\mathcal{L}\left(\boldsymbol{c}\left(\boldsymbol{\pi}\right), \boldsymbol{a}\right):=\int & \left\{\sum_i \lambda_i\left[u\left(c_i\left(\boldsymbol{\pi}\right)\right)-k e_i\right]\right. \\
& -\mu\left(\boldsymbol{\pi}\right)\left[\sum_i c_i\left(\boldsymbol{\pi}\right)-\sum_i \pi_i\right] \frac{1}{\phi^{\boldsymbol{\pi}}\left(\boldsymbol{\pi} \mid \boldsymbol{a}\right)} \\
& -\sum_i \psi_i\left[u\left(c_i\left(\boldsymbol{\pi}\right)\right) \frac{\phi_{e_i}^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i\right) \prod_{j \neq i} \phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j\right)}{\phi^{\boldsymbol{\pi}}\left(\boldsymbol{\pi} \mid \boldsymbol{a}\right)}-v^{\prime}\left(e_i\right)\right] \\
& \left.-\sum_i \sum_q \xi_{i q}\left[y_{z^q}\left(\boldsymbol{a}_i\right)-p^q\right] \frac{1}{\phi^{\boldsymbol{\pi}}\left(\boldsymbol{\pi} \mid \boldsymbol{a}\right)}\right\} \phi^{\boldsymbol{\pi}}\left(\boldsymbol{\pi} \mid \boldsymbol{a}\right) \mathrm{d} \boldsymbol{\pi}
\end{aligned}$$

Then, the first-order condition for $c_i\left(\boldsymbol{\pi}\right)$ reads

$$\lambda_i u^{\prime}\left(c_i^*\left(\boldsymbol{\pi}\right)\right)-\frac{\mu\left(\boldsymbol{\pi}\right)}{\phi^{\boldsymbol{\pi}}\left(\boldsymbol{\pi} \mid \boldsymbol{a}^*\right)}-\psi_i u^{\prime}\left(c_i^*\left(\boldsymbol{\pi}\right)\right) \frac{\phi_{e_i}^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right) \prod_{j \neq i} \phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j^*\right)}{\phi^{\boldsymbol{\pi}}\left(\boldsymbol{\pi} \mid \boldsymbol{a}^*\right)}=0. \tag{17}$$

By independence, $\phi^{\boldsymbol{\pi}}\left(\boldsymbol{\pi} \mid \boldsymbol{a}\right)=\phi^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i\right) \prod_{j \neq i} \phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j\right)$. Hence, Equation (17) boils down to

$$\lambda_i u^{\prime}\left(c_i^*\left(\boldsymbol{\pi}\right)\right)-\frac{\mu\left(\boldsymbol{\pi}\right)}{\phi^{\boldsymbol{\pi}}\left(\boldsymbol{\pi} \mid \boldsymbol{a}^*\right)}-\psi_i u^{\prime}\left(c_i^*\left(\boldsymbol{\pi}\right)\right) \Lambda_i\left(\pi_i \mid \boldsymbol{a}_i^*\right)=0,$$

Combining this with the first-order condition for $c_j(\boldsymbol{\pi})$ delivers Equation (27). $\square$

Proof of Claim 1. Perfect sharing requires Equation (27) to be constant across profit realizations. Suppose this is true; i.e.,

$$\frac{\lambda_j + \psi_j \Lambda_j\left(\pi_j \mid \boldsymbol{a}_j^*\right)}{\lambda_i + \psi_i \Lambda_i\left(\pi_i \mid \boldsymbol{a}_i^*\right)} = r_{ij}, \tag{18}$$

where r_{ij} is a constant. Rearrange Equation (18) to

$$r_{ij}\psi_i \Lambda_i\left(\pi_i \mid \boldsymbol{a}_i^*\right) - \psi_j \Lambda_j\left(\pi_j \mid \boldsymbol{a}_j^*\right) = \lambda_j - r_{ij}\lambda_i := \widehat{r}_{ij},$$

where $\widehat{r}_{ij}$ is yet another constant. Multiply both sides of the previous equation by $\phi^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right)$ to obtain

$$r_{ij}\psi_i \phi_{e_i}^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right) - \psi_j \frac{\phi_{e_j}^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j^*\right)}{\phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j^*\right)} \phi^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right) = \widehat{r}_{ij}\phi^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right).$$

Integrate over π_i using the fact that $\int \phi^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right) = 1$ to get

$$r_{ij}\psi_i \int \phi_{e_i}^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right) \mathrm{d}\pi_i - \psi_j \frac{\phi_{e_j}^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j^*\right)}{\phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j^*\right)} = \widehat{r}_{ij}.$$

Next, multiply both sides of the previous equations by $\phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j^*\right)$, integrate over π_i, and use the fact that $\int \phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j^*\right) = 1$ to obtain

$$r_{ij}\psi_i \int \phi_{e_i}^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right) \mathrm{d}\pi_i - \psi_j \int \phi_{e_j}^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j^*\right) \mathrm{d}\pi_j = \widehat{r}_{ij}.$$

Notice that $\int \phi_{e_i}^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right) \mathrm{d}\pi_i = \int \phi_{e_j}^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j^*\right) \mathrm{d}\pi_j = 0$, since $\int \phi^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right) = \int \phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j^*\right) = 1$. Hence, it must be the case that

$$\widehat{r}_{ij} = \lambda_j - r_{ij}\lambda_i = 0.$$

This is true if and only if $r_{ij} = \frac{\lambda_j}{\lambda_i}$. Combining this last observation with Equation (18), one gets

$$\frac{\lambda_j + \psi_j \Lambda_j\left(\pi_j \mid \boldsymbol{a}_j^*\right)}{\lambda_i + \psi_i \Lambda_i\left(\pi_i \mid \boldsymbol{a}_i^*\right)} = \frac{\lambda_j}{\lambda_i};$$

i.e.,

$$\lambda_i \psi_j \Lambda_j \left(\pi_j \mid \boldsymbol{a}_j^*\right) = \lambda_j \psi_i \Lambda_i \left(\pi_i \mid \boldsymbol{a}_i^*\right). \tag{19}$$

Suppose $\psi_i, \psi_j \neq 0$ (otherwise perfect risk-sharing would trivially obtain). Next, I show that Equation (19) cannot hold for each $\boldsymbol{\pi}$. To see this, pick $(\pi_j, \boldsymbol{\pi}_{-j}) = (\widehat{\pi}_j, \boldsymbol{\pi}_{-j})$. Equation (19) implies that

$$\Lambda_i \left(\pi_i \mid \boldsymbol{a}_i^*\right) = \frac{\lambda_i \psi_j}{\lambda_j \psi_i} \Lambda_j \left(\widehat{\pi}_j \mid \boldsymbol{a}_j^*\right).$$

Next, pick $(\pi_j, \boldsymbol{\pi}_{-j}) = \left(\widehat{\pi}'_j, \boldsymbol{\pi}_{-j}\right)$, with $\widehat{\pi}_j \neq \widehat{\pi}'_j$. Since Equation (19) holds for each $\boldsymbol{\pi}$, it must be the case that

$$\frac{\lambda_i \psi_j}{\lambda_j \psi_i} \Lambda_j \left(\widehat{\pi}_j \mid \boldsymbol{a}_j^*\right) = \Lambda_i \left(\pi_i \mid \boldsymbol{a}_i^*\right) = \frac{\lambda_i \psi_j}{\lambda_j \psi_i} \Lambda_j \left(\widehat{\pi}'_j \mid \boldsymbol{a}_j^*\right).$$

Given that the choices of $\widehat{\pi}_j$ and $\widehat{\pi}'_j$ were totally arbitrary, I conclude that $\Lambda_j \left(\pi_j \mid \boldsymbol{a}_j^*\right)$ must be a constant function of π_j. Hence, it must be the case that

$$\phi_{e_j}^{\pi_j} \left(\pi_j \mid \boldsymbol{a}_j^*\right) = w_j \phi^{\pi_j} \left(\pi_j \mid \boldsymbol{a}_j^*\right),$$

for some constant w_j. This is a first-order linear differential equation in e_i. The solution to this equation is given by

$$\phi^{\pi_j} \left(\pi_j \mid \boldsymbol{a}_j^*\right) = \frac{1}{\exp\left\{\int_0^E w_j \, \mathrm{d}e_i\right\}} \int_0^E \exp\left\{\int_0^E w_j \, \mathrm{d}e_i\right\} 0 \, \mathrm{d}e_i = 0.$$

This contradicts Equation (23). $\square$

Proof of Claim 6. Applying a change of variables from $\boldsymbol{\pi}$ to $\boldsymbol{\varepsilon}$ and assuming that the optimal sharing contract is differentiable, one can write the first-order condition for effort as

$$\int u' \left(c_i^* \left(\boldsymbol{\pi}\right)\right) \frac{\partial c_i^* \left(\boldsymbol{\pi}\right)}{\partial \pi_i} y_e \left(\boldsymbol{a}_i^*\right) \mathrm{d}\Phi^{\boldsymbol{\varepsilon}} \left(\boldsymbol{\varepsilon}\right) = k.$$

This can be rewritten as

$$y_e \left(\boldsymbol{a}_i^*\right) = \frac{k}{\int u' \left(c_i^* \left(\boldsymbol{\pi}\right)\right) \frac{\partial c_i^* (\boldsymbol{\pi})}{\partial \pi_i} \mathrm{d}\Phi^{\boldsymbol{\varepsilon}} \left(\boldsymbol{\varepsilon}\right)} := p \left(c_i^* \left(\boldsymbol{\pi}\right)\right).$$

By the law of supply, e_i^* is strictly decreasing in $p\left(c_i^*\left(\boldsymbol{\pi}\right)\right)$. Finally, notice that $p\left(c_i^*\left(\boldsymbol{\pi}\right)\right)$ is increasing in $\frac{\partial c_i^*(\boldsymbol{\pi})}{\partial \pi_i}$. □

Proof of Theorem 2. By Claim 6, e_i^* is increasing in $\frac{\partial c_i^*(\boldsymbol{\pi})}{\partial \pi_i}$. Notice that $\frac{\partial c_i^*(\boldsymbol{\pi})}{\partial \pi_i}$ affects $p\left(c_i^*\left(\boldsymbol{\pi}\right)\right)$, but not the prices of the other inputs. Hence,

$$\frac{\partial z_i^{q*}\left(p\left(c_i^*\left(\boldsymbol{\pi}\right)\right),\boldsymbol{p}\right)}{\partial \frac{\partial c_i^*(\boldsymbol{\pi})}{\partial \pi_i}} = \frac{\partial z_i^{q*}\left(p\left(c_i^*\left(\boldsymbol{\pi}\right)\right),\boldsymbol{p}\right)}{\partial p\left(c_i^*\left(\boldsymbol{\pi}\right)\right)} \frac{\partial p\left(c_i^*\left(\boldsymbol{\pi}\right)\right)}{\partial \frac{\partial c_i^*(\boldsymbol{\pi})}{\partial \pi_i}}.$$

□

B Contrast Estimator

In this appendix, I describe the relationship between the risk-sharing contract specified in Equation (3) and the well-known contrast estimator ([Suri, 2005]). Consider the following regression equation:

$$c_{ivt} - \overline{c}_{vt} = \delta^W \left(\pi_{ivt} - \overline{\pi}_{vt}\right) + \eta_{ivt}, \tag{20}$$

where c_{ivt} and π_{ivt} are household i's consumption and income in village v and period t, and $\overline{c}_{vt}$ and $\overline{\pi}_{vt}$ are average consumption and income in village v and period t. [Suri, 2005] refers to δ^W as the within-estimator. Assume that the village is a closed economy with no saving technology. Then, the accounting identity $\overline{c}_{vt} = \overline{\pi}_{vt}$ trivially holds, and Equation (20) can be rewritten as follows:

$$c_{ivt} = \delta^W y_{ivt} + \left(1 - \delta^W\right) \overline{y}_{vt} + \eta_{ivt}. \tag{21}$$

Equation (21) makes it clear that sharing rule α, as defined in Equation (3), theoretically coincides with δ^W when the village is a closed economy with no saving technology.

Next, consider the following regression equation:

$$\overline{c}_{vt} = \delta^B \overline{\pi}_{vt} + \nu_{vt}.$$

[Suri, 2005] refers to δ^B as the between-estimator, and defines the contrast estimator as follows:

$$\delta := 1 - \frac{\delta^W}{\delta^B}.$$

Note that if the village is a closed economy with no saving technology then $\delta^B = 1$. Thus, in this case, $\delta = 1 - \delta^W = 1 - \alpha$, and Equation (3) can be rewritten as follows:

$$c_{ivt} = \delta^W \pi_{ivt} + \delta \overline{\pi}_{vt}.$$

C A Model of Endogenous of Risk-Sharing

In this appendix, I extend the model outlined in Section 2 by not constraining risk-sharing to be linear. In this way, I can study the joint determination of insurance, effort, and input choices.

As before, consider n household-farms, each of which chooses an action, $\boldsymbol{a}_i := (e_i, \boldsymbol{z}_i)$, which is combined with an idiosyncratic productivity shock, ε_i, to generate a random output, $y_i = y(\boldsymbol{a}_i) + \varepsilon_i$. Let

$$\begin{aligned} \pi_i := \pi(\boldsymbol{a}_i, \varepsilon_i) &:= y_i - \boldsymbol{p}\boldsymbol{z}_i \\ &= y(\boldsymbol{a}_i) + \varepsilon_i - \boldsymbol{p}\boldsymbol{z}_i \end{aligned} \tag{22}$$

be i's profit. Denote by Φ^{ε_i} and ϕ^{ε_i} the cumulative distribution function (CDF) and the probability density function (PDF) of ε_i. Letting $\widehat{\pi}_i$ be a realization of π_i, the CDF of π_i conditional on $\boldsymbol{a}_i$ is given by $\Phi^{\pi_i}(\widehat{\pi}_i \mid \boldsymbol{a}_i) := \Pr(\pi_i \leq \widehat{\pi}_i)$. This CDF can be calculated as follows:

$$\begin{aligned} \Phi^{\pi_i}(\widehat{\pi}_i \mid \boldsymbol{a}_i) &= \Pr(\pi(\boldsymbol{a}_i; \varepsilon_i) \leq \widehat{\pi}_i) \\ &= \Pr(\varepsilon_i \leq \widehat{\pi}_i - y(\boldsymbol{a}_i) + \boldsymbol{p}\boldsymbol{z}_i) \\ &= \Phi^{\varepsilon_i}(\widehat{\pi}_i - y(\boldsymbol{a}_i) + \boldsymbol{p}\boldsymbol{z}_i) \\ &= \int_{-\infty}^{\widehat{\pi}_i - y(\boldsymbol{a}_i) + \boldsymbol{p}\boldsymbol{z}_i} \phi^{\varepsilon_i}(\varepsilon_i)\, \mathrm{d}\varepsilon_i. \end{aligned} \tag{23}$$

This is a 'parametrized distribution representation' of profit. This representation highlights that different actions give rise to different distributions of profit. It turns out to be analytically convenient to work with both the parametrized distribution representation of profit and its primitive 'state-space representation,' given in Equation (22).[20] Given Equation (23), one can write

$$\begin{aligned}
\phi^{\pi_i}\left(\widehat{\pi}_i \mid \boldsymbol{a}_i\right) &= \frac{\partial \Phi^{\pi_i}\left(\widehat{\pi}_i \mid \boldsymbol{a}_i\right)}{\partial \widehat{\pi}_i} \\
&= \frac{\mathrm{d}}{\mathrm{d}\widehat{\pi}_i} \int_{-\infty}^{\widehat{\pi}_i - y(\boldsymbol{a}_i) + \boldsymbol{p}\boldsymbol{z}_i} \phi^{\varepsilon_i}\left(\varepsilon_i\right) \mathrm{d}\varepsilon_i \\
&= \phi^{\varepsilon_i}\left(\widehat{\pi}_i - y\left(\boldsymbol{a}_i\right) + \boldsymbol{p}\boldsymbol{z}_i\right).
\end{aligned}$$

Throughout the paper, I assume that ϕ^{ε_i} is differentiable.

Let $\boldsymbol{\pi} := \left(\pi_i\right)_i$ be a profile of profits. The consumption received by i when $\boldsymbol{\pi}$ realizes is denoted $c_i\left(\boldsymbol{\pi}\right)$. The feasibility constraint dictates that

$$\sum_{i \in N} c_i\left(\boldsymbol{\pi}\right) \leq \sum_{i \in N} \pi_i,$$

for each $\boldsymbol{\pi}$. Household i's utility is

$$u\left(c_i\left(\boldsymbol{\pi}\right)\right) - \kappa e_i,$$

where u is twice-continuously differentiable, strictly increasing, and strictly concave.[21]

Let $\boldsymbol{a} := \left(\boldsymbol{a}_i\right)_i$ and $\Phi^{\boldsymbol{\pi}}\left(\boldsymbol{\pi} \mid \boldsymbol{a}\right) := \prod_i \Phi^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i\right)$. This is the cumulative distribution function of $\boldsymbol{\pi}$, because profits are independent between households once conditioning on actions taken. Finally, let $\boldsymbol{c}\left(\boldsymbol{\pi}\right) := \left(c_i\left(\boldsymbol{\pi}\right)\right)_i$ be the sharing contract. An allocation is a pair $\left(\boldsymbol{c}\left(\boldsymbol{\pi}\right), \boldsymbol{a}\right)$.

Full Information. Assume that $\boldsymbol{a}$ is observed by the planner. In this case, there are no information frictions, so the planner can implement any action at no cost. Formally, the

[20] See, e.g., [Conlon, 2009] for a discussion of the differences between state-space and parametrized distribution representations.

[21] The assumption that expected utility is separable in consumption and effort is standard in the moral hazard literature.

planner's problem is

$$\begin{aligned} &\max_{\boldsymbol{c}(\boldsymbol{\pi}),\boldsymbol{a}} \sum_{i\in N} \lambda_i \left\{ \mathbb{E}_{\boldsymbol{\pi}} \left[u\left(c_i\left(\boldsymbol{\pi}\right)\right) \mid \boldsymbol{a} \right] - k e_i \right\}, \\ &\text{subject to } \sum_{i\in N} c_i\left(\boldsymbol{\pi}\right) = \sum_{i\in N} \pi_i, \ \forall \boldsymbol{\pi}. \end{aligned} \tag{24}$$

Notice that the feasibility constraint holds with equality. This is without loss of generality, as the constraint must bind at a solution to the problem. Notice also that there are no participation constraints. Again, this is without loss of generality, as the planner is benevolent. The following proposition characterizes the optimal sharing contract under full information. The proposition shows that the first-order conditions for households' consumptions imply that the ratio of marginal utilities across any two households is constant across profit realizations. This is Borch's rule; i.e., the condition for perfect risk-sharing (when the solution is interior).

Proposition 1. *Under full information there is perfect risk-sharing.*

The following claim characterizes the an optimal action profile.

Claim 5. *Let $(\boldsymbol{c}^*(\boldsymbol{\pi}), \boldsymbol{a}^*)$ be a solution to Problem (24). At $\boldsymbol{a}^*$, (i) the social expected marginal benefit of effort is equal to its private marginal cost, and (ii) the marginal product of any intermediate q is equal to its price.*

The intuition behind this corollary is provided by Samuelson's rule for the optimal provision of public goods. This rule states that, at an optimum, the social marginal benefit of a public good equals the marginal cost of providing it. The key is to notice that when households share profits, so that there exist households i and j such that $c_i(\boldsymbol{\pi})$ is not constant in π_i, effort gives rise to an externality: a change in effort on the part of j directly affects i's consumptions. On the other hand, the condition for the optimal use of intermediate q is the standard optimality condition for a market-provided private good. The reason is that, contrary to effort, intermediates do not give rise to externalities. This asymmetry between effort and intermediates follows from the assumptions that (i) households share profits, so that they share revenues as much as the monetary costs of inputs, and (ii) effort does not enter the monetary costs of inputs, as there is no market for effort (i.e., each household can only supply effort to its own agricultural business). Hence, the impact of the sharing contract on the private marginal benefit of an intermediate cancels out with its impact on the private marginal cost of that intermediate, while this is not true for effort, for which the sharing contract only decreases private marginal benefits while leaving unaltered private marginal costs.

Private Information. Next, assume that the action taken by i and the shock it receives are private to i. In this case, profits are publicly observable, noisy signals of actions taken. After observing the signals, the planner collects the profits realized and redistributes them to the households according to the sharing contract he designs. The planner takes into account that the households non-cooperatively choose an action profile given the sharing contract. Formally, the planner's problem is

$$\begin{aligned} &\max_{\boldsymbol{c}(\boldsymbol{\pi}),\boldsymbol{a}} \sum_{i\in N} \lambda_i \left\{\mathbb{E}_{\boldsymbol{\pi}}\left[u\left(c_i\left(\boldsymbol{\pi}\right)\right) \mid \boldsymbol{a}\right] - ke_i\right\} \\ &\text{subject to } \sum_i c_i\left(\boldsymbol{\pi}\right) = \sum_i \pi_i, \ \forall \boldsymbol{\pi}, \\ &\text{and } \boldsymbol{a}_i \in \arg\max_{\widetilde{\boldsymbol{a}}_i} \mathbb{E}_{\boldsymbol{\pi}}\left[u\left(c_i\left(\boldsymbol{\pi}\right)\right) \mid \widetilde{\boldsymbol{a}}_i, \boldsymbol{a}_{-i}\right] - k\widetilde{e}_i, \ \forall i. \end{aligned} \tag{25}$$

The difference between (25) and (24) is that in the private information regime an allocation has to simultaneously satisfy n incentive-compatibility (IC) constraints. These constraints essentially define a pure-strategy Nash equilibrium: at $\boldsymbol{a}$, no household wants to deviate when it correctly anticipates the other households' actions. The set of IC constraints is a complicated object, as it comprises of a set of intertwined optimization problems. Moreover, there might exist more than one Nash equilibrium (or even none).

Many papers in the principal-agent literature dealing with similar contracting problems have relied on the first-order approach (FOA), by which the agent's IC constraint is replaced by its first-order conditions. The optimal contract is then easily derived. The literature ([Rogerson, 1985] and [Jewitt, 1988]) has then focused on providing sufficient conditions under which the FOA is valid. My problem is different from the canonical principal-agent problem as there are n agents and each of them is choosing a multidimensional action. To gain intuition, it is worthwhile to set the stage by characterizing the optimal sharing contract under the assumption that the FOA is valid. More formally, I begin by considering a relaxed version of Problem (25), in which the IC constraints are replaced with the requirement that the action chosen by each household be a stationary point, given the actions chosen by the other households and the sharing contract. The key assumption is that a solution to the relaxed version of the problem is also a solution to Problem (25).[22]

Assumption 1. *Let $\left(\boldsymbol{c}^{*}\left(\boldsymbol{\pi}\right), \boldsymbol{a}^{*}\right)$ be a solution to the following relaxed version of Problem (25):*

$$\begin{aligned}
&\max_{\boldsymbol{c}(\boldsymbol{\pi}),\boldsymbol{a}} \sum_{i \in N} \lambda_i \left\{\mathbb{E}_{\boldsymbol{\pi}}\left[u\left(c_i\left(\boldsymbol{\pi}\right)\right) \mid \boldsymbol{a}\right] - k e_i\right\} \\
&\textit{subject to } \sum_i c_i\left(\boldsymbol{\pi}\right) = \sum_i \pi_i, \ \forall \boldsymbol{\pi}, \\
&\int u\left(c_i\left(\boldsymbol{\pi}\right)\right) \phi^{\pi_i}_{e_i}\left(\pi_i \mid \boldsymbol{a}_i\right) \prod_{j \neq i} \phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j\right) \mathrm{d}\boldsymbol{\pi} = k, \forall i, \\
&\int u\left(c_i\left(\boldsymbol{\pi}\right)\right) \phi^{\pi_i}_{z_i^q}\left(\pi_i \mid \boldsymbol{a}_i\right) \prod_{j \neq i} \phi^{\pi_j}\left(\pi_j \mid \boldsymbol{a}_j\right) \mathrm{d}\boldsymbol{\pi} = 0, \ \forall i, \forall q.
\end{aligned} \tag{26}$$

This solution is a solution to Problem (25).

The following proposition characterizes the optimal sharing rule under private information when Assumption 1 holds.

Proposition 2. *Suppose that Assumption 1 holds. Then, the optimal sharing rule is pinned down by*

$$\frac{u'\left(c_i^*\left(\boldsymbol{\pi}\right)\right)}{u'\left(c_j^*\left(\boldsymbol{\pi}\right)\right)} = \frac{\lambda_j + \psi_j \Lambda_j\left(\pi_j \mid \boldsymbol{a}_j^*\right)}{\lambda_i + \psi_i \Lambda_i\left(\pi_i \mid \boldsymbol{a}_i^*\right)}, \tag{27}$$

for each $i, j \in N$, where ψ_i is the Lagrange multiplier associated to household i's first-order condition for effort and

$$\Lambda_i\left(\pi_i \mid \boldsymbol{a}_i^*\right) := \frac{\phi^{\pi_i}_{e_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right)}{\phi^{\pi_i}\left(\pi_i \mid \boldsymbol{a}_i^*\right)}.$$

Equation (27) is a modified Borch rule. In particular, if $\psi_i \neq 0$ or $\psi_j \neq 0$, then the ratio of marginal utilities between households may vary across profit realizations. The wedge between

[22] Most likely, it is not so useful to give general conditions for the validity of the FOA, as these would probably not be conditions that generalize the specific assumptions of the model in Section 2. I would be better off showing that with quadratic utility (which implies mean-variance expected utility) and Lagrange shocks, the first-order approach is valid, as suggested in [Wang, 2013]. In any case, see Appendix E for a first attempt in providing general conditions for the validity of Assumption 1.

Equation (27) and the Borch rule is designed by the planner to take into account the effect of the sharing contract on the incentives to exert effort. On the other hand, the planner does *not* take into account the effect of the sharing contract on the use of agricultural inputs. This follows because the households are sharing profits—the revenues of production minus the monetary costs of agricultural inputs. Profit sharing implies that the effect of the sharing contract on the marginal benefit of an agricultural input is compensated by the effect of the contract on the marginal cost of that input; i.e., the sharing contract does not distort the incentives to use inputs purchased in the market. Finally, the wedge between Equation (27) and the Borch rule implies that risk-sharing is generally imperfect under private information, as shown in the following corollary.

Corollary 1. *Under private information risk-sharing is imperfect.*

In the following, I consider the interaction between insurance, effort choices, and use of agricultural inputs. To do so, I focus on the implementation of a given action profile, and analyze how actions change when the sharing contract is perturbed. A more satisfying approach would be to jointly deriving an optimal action profile and the optimal sharing contract implementing it as a function of the parameters of the model—the utility cost of effort, the price of fertilizer, the variance of the production shock, and so on. Then, one could analyze how exogenous changes in these parameters jointly affect the sharing contract and the action profile, and thus keep track of the relationship between risk-sharing and actions. I choose to follow the first approach because jointly deriving an optimal action in addition to the optimal contract that implements it is typically a very complex problem. Analyzing how actions change when the sharing contract is perturbed allows for significant tractability and is useful for practical applications.[23] To gain tractability, consider the case in which the optimal sharing contract is differentiable,[24] and define the slope of the contract at $\boldsymbol{\pi}$ for household i as

$$\frac{\partial c_i^*(\boldsymbol{\pi})}{\partial \pi_i}.$$

[23]In fact, most of the papers in both the theoretical and applied literatures on the principal-agent problem focus exclusively or predominantly on implementing a given action ([Edmans and Gabaix, 2011]).

[24]This approach is not entirely satisfactory, as $\boldsymbol{c}^*(\boldsymbol{\pi})$ is an endogenous object which was computed by means of point-wise maximization; hence, there is no a priori reason to expect $\boldsymbol{c}^*(\boldsymbol{\pi})$ to be differentiable. While not being rigorous, this approach is common practice (see e.g. Appendix B in [Attanasio and Pavoni, 2011]).

Intuitively, the slope of the contract measures the responsiveness of consumption to income. The smaller is the slope of the contract at $\boldsymbol{\pi}$, the higher is the insurance it provides at that profit realization. If the sharing contract is linear, than its slope is constant. Moreover, as shown in Subsection B, in a closed economy with no savings, the slope of a linear sharing contract coincides with the within estimator, δ^W. The following claim generalizes the intuition provided by the model of exogenous risk-sharing in Section 2 by showing that, when the optimal sharing contract is differentiable, making the contract steeper (i.e., decreasing insurance) for household i induces the household to exert more effort.

Claim 6. *Assume that the optimal sharing contract is differentiable. The higher is the slope of the contract at any $\boldsymbol{\pi}$, the higher is the effort provided.*

This result is based on the fact that the 'effective' price of effort is decreasing in the slope of the contract. Next, I prove the main theorem, which extends the results of Theorem 1 to the case in which risk-sharing is endogenous.

Theorem 2. *Let $(\boldsymbol{c}^*(\boldsymbol{\pi}), \boldsymbol{a}^*)$ be a solution to the planner's problem under private information. Suppose that e_i and z_i^q are substitutes at $(p(c_i^*(\boldsymbol{\pi})), \boldsymbol{p})$; i.e.,*

$$\frac{\partial z_i^{q*}(p(c_i^*(\boldsymbol{\pi})), \boldsymbol{p})}{\partial p(c_i^*(\boldsymbol{\pi}))} > 0.$$

Then,

$$\frac{\partial z_i^{q*}(p(c_i^*(\boldsymbol{\pi})), \boldsymbol{p})}{\partial \frac{\partial c_i^*(\boldsymbol{\pi})}{\partial \pi_i}} < 0.$$

The signs of the latter two inequalities are reversed if e_i and z_i^q are complements at $(p(c_i^*(\boldsymbol{\pi})), \boldsymbol{p})$.

This theorem generalizes Theorem 1. In particular, it makes it clear that all of the results obtained in Section 2 can be obtained in a model of endogenous risk-sharing, in which the optimal sharing contract is differentiable. In the latter case, changing α would amount to making the sharing contract steeper.

C.1 Mean-Variance Expected Utility

Problem (25) is a complicated one, as the n incentive-compatibility constraints define a pure-strategy Nash equilibrium. In this subsection, I show that if households have mean-variance expected utility and the optimal sharing rule is linear, Problem (25) can be greatly simplified as each household's optimal action is independent of what the other households do.[25] In order to justify mean-variance expected utility, one can assume that the utility from consumption is quadratic; i.e.,

$$u\left(c_i\left(\boldsymbol{\pi}\right)\right) = c_i\left(\boldsymbol{\pi}\right) - \frac{\rho}{2} c_i\left(\boldsymbol{\pi}\right)^2.$$

In this case, the expected utility of consumption takes a mean-variance specification, independently of the distribution of $c_i\left(\boldsymbol{\pi}\right)$. Alternatively, if the sharing contract is indeed linear and the production shocks are normally distributed,[26] then $c_i\left(\boldsymbol{\pi}\right)$ is also normally distributed, and the households have mean-variance expected utility when their utility from consumption is CARA; i.e.,

$$u\left(c_i\left(\boldsymbol{\pi}\right)\right) = -\exp\left\{-\rho c_i\left(\boldsymbol{\pi}\right)\right\}.$$

Let $\boldsymbol{c}^{\mathrm{FB}}$ be the optimal sharing rule under full information (superscript FB stands for 'first best'). The following claim holds:

Claim 7. *If the households have quadratic or CARA utility from consumption and $\lambda_i = \lambda_j$, for each $i, j \in N$, then $c_i^{\mathrm{FB}}\left(\boldsymbol{\pi}\right) = \overline{\pi}$.*

Proof. Proposition 1 shows that, under full information, the optimal sharing rule is pinned down by the Borch rule:

$$\frac{u'\left(c_i^{\mathrm{FB}}\left(\boldsymbol{\pi}\right)\right)}{u'\left(c_j^{\mathrm{FB}}\left(\boldsymbol{\pi}\right)\right)} = \frac{\lambda_j}{\lambda_i},$$

If agents have quadratic utility from consumption and $\lambda_i = \lambda_j$, this boils down to

$$\frac{1 - \gamma c_i^{\mathrm{FB}}\left(\boldsymbol{\pi}\right)}{1 - \gamma c_j^{\mathrm{FB}}\left(\boldsymbol{\pi}\right)} = 1.$$

[25] I.e., in this case, the n incentive-compatibility constraints define a dominant strategy equilibrium.

[26] This argument assumes that the sharing contract is linear. In fact, if I were to assume the the sharing contract is chosen by the planner, then this argument would break, as Mirrlees Put reference famously shows that in a CARA-normal principal-agent model (was the principal risk-neutral?), an optimal sharing contract does not exist.

Hence, it must be the case that $c_i^{\mathrm{FB}}(\boldsymbol{\pi}) = c_j^{\mathrm{FB}}(\boldsymbol{\pi})$, for each $j \neq i$. Using the feasibility constraint, this implies that $c_i^{\mathrm{FB}}(\boldsymbol{\pi}) = \frac{\sum_j \pi_j}{n}$. It is easy to see that this corresponds to the case in which the planner uses linear contracts, with $\alpha_{ij} = \frac{1}{n}$. With CARA utility,

$$c_i^{\mathrm{FB}}(\boldsymbol{\pi}) = \frac{\sum_{j \in N} \pi_j}{n} + \frac{1}{n\rho} \log\left(\frac{\prod_{j \neq i} \lambda_j}{\lambda_i^{n-1}}\right)$$

(see [Ambrus et al., 2017]). Hence, the result follows when $\lambda_i = \lambda_j$, for each $i, j \in N$. $\square$

Mean-variance expected utility is particularly tractable because household i's choices are independent of the other households' choices when the sharing contract is linear. I proceed by demonstrating this result under the assumption that the sharing contract is linear. Then, I provide conditions which ensure that the optimal contract is indeed linear [WORK IN PROGRESS]. Let $\boldsymbol{c}^{\mathrm{SB}}$ be the optimal sharing rule under private information (superscript SB stands for 'second best'). Assume that $\boldsymbol{c}^{\mathrm{SB}}$ is linear; i.e.,

$$c_i^{\mathrm{SB}}(\boldsymbol{\pi}) = \sum_{j \in N} \alpha_{ij}^{\mathrm{SB}} \pi_j$$

The following claim proves that when the households have mean-variance expected utility, household i's choices are independent the other households' choices.

Claim 8. *When $\boldsymbol{c}^{\mathrm{SB}}$ is linear and the agents have mean-variance utility, household i's choices are independent of what the other households do.*

Proof. When $\boldsymbol{c}^{\mathrm{SB}}$ is linear and the agents have mean-variance expected utility, household i's problem can be written as

$$\max_{\boldsymbol{a}_i} \sum_{j \in N} \alpha_{ij}^{\mathrm{SB}} \left(y(\boldsymbol{a}_j) - \boldsymbol{p} \cdot \boldsymbol{z}_j + \mu\right) - \frac{\rho}{2} \sum_{j \in N} \alpha_{ij}^{\mathrm{SB}^2} \sigma^2 - v(e_i).$$

The objective function is continuously differentiable and jointly concave in e_i and z_i^q. Hence, the maximization problem is a concave program and the first-order conditions pin down an interior solution. The first-order conditions for e_i and z_i^q are given by

$$\alpha_{ii}^{\mathrm{SB}} y_e(\boldsymbol{a}_i^*) = k$$

and

$$y_{z^q}\left(\boldsymbol{a}_i^*\right) = p_q,$$

respectively. Notice that these conditions are independent of $\boldsymbol{a}_j$, for $j \neq i$. □

The key here is to understand when a linear sharing contract would be optimal. **Any suggestions?**

D Complements and Substitutes

[WORK IN PROGRESS]

E Justifying the FOA

[WORK IN PROGRESS]

F Quadratic Disutility of Effort

Assume that the disutility of effort is ke_i^2. In this case, the marginal rate of substitution of effort for consumption for household i is

$$\frac{\mathrm{d}c_i\left(\alpha\right)}{\mathrm{d}e_i} = \frac{\kappa_i e_i}{\rho c_i\left(\alpha\right)}.$$

When the disutility of effort is ke_i^2, one should estimate

$$\log\left(\frac{f_{it}}{e_{it}}\right) = \sigma\log\left(\kappa_i\right) + \sigma\log\left(e_{it}\right) - \sigma\log\left(1 - \frac{n_{vt}-1}{n_{vt}}\alpha_{vt}\right) - \sigma\log\left(p_{it}\right) + \epsilon_{it},$$

instead of Equation (13). In econometric terms, the previous equation reads

$$\log\left(\frac{f_{it}}{e_{it}}\right) = \varphi_i + \phi_{vt} + \beta_1\log\left(p_{it}\right) + \beta_2\log\left(p_{it}\right) + \epsilon_{it},$$

Table 11 reports the OLS estimates of the previous regression. Importantly, $\widehat{\beta}_1$ and $\widehat{\beta}_2$ are

Table 11: Another Structural Estimation

Dep. variable: $\log\left(\frac{f_{it}}{e_{it}}\right)$	$\widehat{\beta}$ (s.e.)
$\log(e_{it})$	-.4818***
	(.0172)
$\log(p_{it})$	-.5719*
	(.3001)
Household fixed effects	Yes
Village-month fixed effects	Yes
R-squared	0.388
Observations	6,481

Notes: OLS regressions of log effort and log fertilizer price on log fertilizer over effort. Standard errors are robust.

statistically equal, as expected (p-value: 0.76). In this case, the average marginal cost of effort is approximately 19; hence, the average marginal rate of substitution of effort for consumption is approximately 0.1. While still quite low, the average marginal rate of substitution of effort for consumption has the same order of magnitude as the lower bound of the range of wages for agricultural workers given in [Indian Labour Bureau, 2010].

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