The Impact of Monitoring Technologies on Contracts and Employee Behavior: Experimental Evidence from Kenya's Matatu Industry

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Abstract

Agency theory suggests that alleviating moral hazard will help firms design more efficient employment contracts, and align employees interests more closely with their own. We use a randomized control trial to investigate the extent to which monitoring technologies can resolve moral hazard and help firms design more efficient contracts that increase profits and encourage business growth. To this end, we introduce a monitoring device that tracks real-time vehicle location, daily productivity, and safety statistics of 250 minibuses operating in Nairobi. We randomize whether minibus owners have access to the information provided by the device through a novel mobile app. The information we provide allows treatment owners to observe a more precise signal of driver effort, the amount of revenue they collect in fares, and the extent to which the driver engages in risky driving (namely off-route driving). We find that providing information to vehicle owners allows them to modify the terms of their contracts with drivers by decreasing the rental price they demand. Employees respond to the device, and the change in the contract, by exerting more effort, decreasing risky behavior that damages the vehicle, and under-reporting revenue by less, leading to an overall increase in firm profitability and making it easier for firms to expand. Finally we investigate whether these gains to the company come at the expense of commuters and passengers safety, which are already at risk in an industry where accidents are common. While we do not find any evidence that conditions deteriorate, they also do not improve despite the detailed information on safe driving we provide. Only by incentivizing drivers through an additional cash-treatment do we see improvements in safety.

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1 Introduction

Firms design contracts to ensure their employees exert the profit-maximizing level of effort. In the presence of moral hazard, however, firms cannot condition the terms of the contract on fundamental dimensions of employee behavior. In seminal asymmetric information models, firms respond to this uncertainty by relying on "second-best" self-enforcing incentive contracts, or coercive measures to align agents interests with their own (Hölmstrom, 1979; Grossman and Hart, 1983; Hart and Holmstrom, 1987; Shapiro and Stiglitz, 1984). In low income countries, employees' lack of capital (limited liability) and weak legal institutions prevent companies from credibly sanctioning bad behavior, potentially increasing the wedge between second-best and optimal contracts (Sappington, 1983). In theory, firms can overcome these frictions by investing in monitoring technologies that reduce information asymmetries and reveal the performance of their workers more accurately (Harris and Raviv, 1979; Hölmstrom, 1979; Hubbard, 2003). In practice, however, the positive impact of these technologies in low-income countries is unclear. The presence of institutional and managerial frictions may limit employers' ability to leverage the additional information to change the contract and employees behavior.

The primary objective of this paper is to determine the impact of moral hazard on labor contracting, productivity, and firm profits and the extent to which improved monitoring eases these frictions. To this end, we implement a randomized control trial where we introduce a novel monitoring device to a subset of firms operating in Kenya's public transit industry. The industry is dominated by thousands of small-scale entrepreneurs that own a few minibuses ("matatus") that run on designated routes. These matatus are the only reliable form of public transportation and serve 70% of Nairobi's four million commuters daily. We recruited 250 minibuses operating along 9 major commuter routes to participate in the study, and randomly selected 125 to be part of the treatment group. The monitoring device was fitted to all the matatus in our sample, but only transmitted data to treatment owners. We developed our own device because available alternatives on the market were either too costly, or not sophisticated enough. Our device records and transmits via a mobile app the location of the vehicle, the number of kilometers driven, and the number of hours the ignition was on. While the owner does not know the number of passengers that boarded the vehicle, they can use this information to monitor drivers' operations throughout the day and gain a more precise estimate for total daily revenue.

The contracting environment we study here is not unique to Kenya or transportation. The dynamics that characterize this space are prevalent in many other settings including agriculture and the service industry. First, employers (owners) cannot observe the amount of revenue the employee (drivers) collects, nor the amount of effort the driver invests. Second, drivers in this setting are from relatively poor households and they cannot afford to walk away without pay on

¹Similar transit systems are present in Mexico (peseros), the Philippines (jeepney's), Indonesia (tuk-tuks), India (rickshaws), and Tanzania (dala-dala's), among others

²We only included owners that had 1 minibus, and who managed their own vehicle in order to focus our analysis on the classic principal-agent relationship

days when total revenue is low, nor can they pay for repairs when the vehicle is damaged (limited liability). Drivers are known to run away from accidents so they can avoid being held accountable by owners or the police. In light of these constraints, firms have overwhelmingly opted for fixed rent contracts (locally referred to as a "target" contract) with limited liability. The owner specifies an amount of revenue that the driver must deliver by the end of the day, net of fuel expenses. According to the strict letter of the contract, the driver should deliver the fixed rent ("target") amount if the revenue they collect exceeds the target (keeping any revenue they earn above it). If the earnings are below the target, the contract stipulates that the driver must hand over all of the revenue they earned.

In order to understand the impact of the device, we adapt a standard principal agent framework to reflect the actual employer-employee relationships within this network. In the absence of a monitoring technology, the model predicts that drivers will engage in a number of behaviors that are sub-optimal from the firm's perspective. First, drivers under-report revenue any day they make slightly above the target or below. In doing so, they can be sure to walk away with slightly more income than they would otherwise. Note that rampant cheating is kept in check by owners who threaten to punish and ultimately fire drivers who are caught under-reporting. Second, drivers under-supply effort on days when they are unlikely to make the target price. On these particular days, drivers know they will not be the residual claimant and reap the benefits of higher effort. Third, drivers engage in more reckless driving than what owners would optimally choose. Reckless driving refers to maneuvers that may damage the vehicle. This includes driving on the shoulder of the road, or veering off the designated route onto roads that are more bumpy and damaging to the vehicle. The driver engages in this behavior because he reaps the benefits in terms of higher revenue, but does not bear any of the downside risk (the limited liability constraint binds).

We model the introduction of the new technology as increasing: 1) the precision of the owner's signal about total revenue, and 2) the probability the owner detects reckless driving. This has implications for the owners' choice of contract and drivers' behavior, which we test in our data. According to the model, the monitoring technology reduces drivers' information rents, and lowers their utility. Owners recognize this and compensate them by reducing the target. Empirically, we find that owners steadily reduce the target throughout the study period. By the last month of the study the target is approximately 4.1 percentage points lower than at baseline. While the downward trend is prominent, the coefficient is not statistically significant, likely because contract change of this magnitude takes time in an environment where social norms are engrained.

The model then predicts that drivers behavior is affected along three key dimensions. First, drivers incentive to lie about the total amount of revenue they collected is reduced. This means that we should see less under-reporting. We confirm this prediction in our data: under-reported revenue falls by 100 shillings (a 12% point decrease). Second, the model predicts that drivers will increase their effort to compensate for the income they lose from less under-reporting. In parallel, as the target falls, drivers also have an incentive to increase their effort because they can become the residual claimant more easily. We capture a precise measure of effort through the tracking

device, which powers on and off with the matatu. We find that the number of hours the vehicle is on the road increases by 1.2 hours per day (a 12 percentage point increase). Finally, we expect the driver to reduce instances of reckless driving because they are more likely to be caught by the owner. We proxy reckless driving by the amount of repair costs the owner incurs. We find these decrease by 200 shillings (40 percentage points) by the last month of the study. We have evidence to suggest this comes from fewer instances of driving on alternate-routes that are bumpy.

Next, we investigate the effect of these changes on firm profitability and worker well-being. We find that profits increase by 40%, which is primarily driven by lower repair costs. These gains in firm profits more than offset the cost of the device, suggesting that a tracking device like the one we designed for this study would be a worthwhile investment if it were available on the market. Owners also report that monitoring their drivers has become significantly easier, and they trust their drivers more, consistent with a reduction in management costs. Does this improved profitability and better management enable business growth? We find that treatment owners have 0.145 more vehicles on average than control owners at endline. This suggests that inadequate monitoring may represent an important barrier to firm growth in low-income countries. Turning next to driver well-being, we find that drivers work more but earn the same salary per hour. Moreover, in a qualitative survey we conducted six months after the experiment concluded, we find that 65% of drivers say it made their job easier (26% said nothing changed). This suggests that the effect of new technologies on worker well-being is nuanced.

These first set of results demonstrate how alleviating moral hazard affects operations within the firm. However, the presence of monitoring devices can also have effects *outside* of the firm, as profit maximizing behavior by a private company may impose negative externalities. In public transportation systems, monitoring technologies are often used to check and limit instances of unsafe driving. Kenya's matatu sector is notorious for it's poor safety standards: drivers often swerve, stop suddenly, and turn sharply in order to collect more passengers. Our monitoring technology also records these four instances and conveys them to owners through a separate tab in the mobile app. It is important to note that these traditional unsafe driving events need not be perfectly correlated with the reckless driving behavior we outlined above. For example, drivers often choose to bypass slow moving traffic by taking rough, unpaved roads that harm the vehicle, but pose no safety danger. Alternatively, driving quickly through crowded pedestrian areas may be unsafe, but it is unlikely to cause significant damage to the vehicle. A priori, it was not clear how the owners would use the safety information we provided. On the one hand, owners may internalize the dangers associated with unsafe driving because they bear the cost if their vehicle gets into an accident (or fined). In that case, they might use the information to improve driving quality. On the other hand, owners may perceive the risk of accidents to be low and disregard this information. If they use the device to incentivize effort, the number of safety violations could worsen over the study period, producing a negative externality for Nairobi's commuters.

Despite all the safety information we provided, the frequency of unsafe driving events flagged by the device do not change significantly, and instances of speeding remain the same. It follows that the gains to firms do not come at the expense of commuters. However, this also suggests that external intervention may be necessary to improve safety. We test the efficacy of one such intervention by providing small cash incentives to drivers conditional on safe driving. This treatment is designed to mirror the actions that a regulatory body could potentially take in this setting.³ Our objective is to determine the effectiveness of an intervention that encourages the employees (drivers) rather than the employers (owners) to internalize the negative externalities produced by the business. We find that the cash incentives meaningfully reduced safety violations committed by drivers, confirming that third party intervention can successfully address these firm externalities. However, these effects did not persist after the removal of the cash incentives, suggesting that further action or permanent regulation is needed to induce long-lasting change.

This paper contributes to three different literatures. First, the paper speaks to the vast theoretical work on principal agent relationships and contract formation. These papers focus on deriving the optimal contract subject to various constraints (Hölmstrom, 1979; Grossman and Hart, 1983; Hart and Holmstrom, 1987; Shapiro and Stiglitz, 1984). Our paper, by contrast, empirically demonstrates how contracts change when these constraints are alleviated by monitoring technologies. Measuring the impact of monitoring is challenging because shirking behavior is hard to detect by design, a firm's decision to monitor is often not random, and data on firm operations are difficult to obtain. There are only two other papers to our knowledge that overcome these limitations. Baker and Hubbard (2003, 2004) investigate how the introduction of onboard diagnostic computers (OBCs) change ownership patterns in the U.S trucking industry. Baker and Hubbard (2004) demonstrate that shipping companies respond to the introduction of OBCs by hiring drivers to operate their vehicles (rather than working with drivers that already own their own trucks). Our paper differs from this existing work in three ways. First, we generate exogenous variation in the usage of monitoring technologies by randomizing which companies receive data from a tracking device. This removes some of the potential concerns associated with the difference-in-difference and IV approaches that are used in Baker and Hubbard (2004). Second, we capture high frequency data on contracts and worker behavior. This allows us to monitor how different dimensions of the contract, and worker performance, change over time. We can then documents the impact on firm profits and worker well-being (salary per hour, hours worked, sense of trust).

Finally, these papers are concentrated in developed countries, and we have reason to believe that the impacts of monitoring could be different in low-income countries. First, management quality is different, and employers might not use the information as effectively (Bloom et al., 2013; Bloom, Sadun, and Van Reenen, 2017). Second, employers face additional frictions that may limit their ability to use the information: law enforcement is weak and limited liability constraints bind. Contrary to Baker and Hubbard (2004), we do not find that ownership patterns change as a result of monitoring, precisely because of existing constraints (the driver cannot afford their own vehicle). Nevertheless, we do find that firms use the technology to change the terms of the contract, and induce their employees to behave in a way that aligns with the firms best interest. There is only

³South Africa's Ethekwini municipality is testing one such intervention in the coming months ((Payet, 2018)).

one other paper to our knowledge that investigates the impact of monitoring technologies within firms in a developing country. de Rochambeau (2018) studies the use of GPS devices by managers in Liberia's long range trucking industry. She finds that monitoring technologies crowd out high performing workers intrinsic motivation.⁴ Our analysis builds on this work by investigating how monitoring technologies alleviate other key dimensions of moral hazard (including reckless driving, and lying about total revenue). We also focus on the impact of these devices on contracts.

Second, our findings add to the literature investigating the barriers to firm growth in low-income countries. Identifying the constraints to firm growth is a question of great policy relevance given their large role in these developing economies. Empirical research on small firm growth has identified three key challenges facing firms: credit constraints, labor market frictions, and managerial deficits (Bloom et al., 2014). Our paper most closely resembles the work on managerial deficits, which refers to the difficulties firms face managing the day to day operations (including financial accounts and inventories), as well as incentivizing and monitoring workers. Most of the work in this space studies the impact of business training programs (Bloom et al., 2013; Bloom, Sadun, and Van Reenen, 2017; McKenzie and Woodruff, 2016; Berge, Bjorvatn, and Tungodden, 2014; de Mel, McKenzie, and Woodruff, 2014; Valvidia, 2012). These interventions provide information about how to manage aspects of the businesses that don't involve employees (maintaining business records, separating finances, inventory, controlling for quality, marketing). In contrast, our paper focuses on the role of moral hazard, and how providing information specifically about employees' behavior can change firm operations. We find monitoring technologies improve firm profits and reduce management costs, which helps treatment firms grow. As prices fall, these technologies are becoming increasingly prevalent, making their impacts important to understand.

Finally, our results on reckless driving and traditional safety metrics contribute to a growing empirical literature on policies that promote compliance with government regulation - in this case with safety regulation. In recent years, international institutions have provided funding, knowledge and technical assistance to build systems aimed at reducing the number of traffic injuries and deaths worldwide (World Bank, 2014).⁵ These efforts are difficult to evaluate because the investments are multi-faceted and typically rolled out across an entire city. One exception is a program that was launched in Kenya, which placed stickers inside Nairobi's matatus to encourage passengers to complain to their drivers about unsafe driving (Habyarimana and Jack, 2015). They find that the intervention reduced accidents by 25-30%. Our intervention complements their approach by asking whether owners and drivers can be incentivized to improve safe driving rather than passengers. We find that owners with access to the monitoring device do not internalize the negative externalities produced by their drivers. We only document improvements in safety when we directly incentivize

⁴Note there are additional studies that document the impacts of monitoring in low-income countries - but they do not focus on employer-employee relationship within the firm. Duflo, Hanna, and Ryan (2012) find that teacher absenteeism in India decreases when their attendance is monitored; Björkman and Svensson (2009) demonstrate that community health workers exert more effort when their performance is scrutinized by the community; and Duflo et al. (2013) find that incentives for third-party auditors can improve their reporting.

⁵According to the Global Status Report on Road Safety, 1.24 million people are killed in traffic accidents each year and 90 percent of these deaths occur in low- and middle-income countries (LMICs)

drivers. This suggests that investments in technologies that monitor unsafe driving may be more effective when combined with incentives directed at those operating the vehicles.

The remainder of this paper is organized as follows. Section 2 discusses Kenya's public transportation system, the prevalence of moral hazard, and the scope for monitoring. Section 3 details the field experiment, and Section 4 reviews the data. We present a simple theoretical framework in Section 5. Section 6 discusses each of our results. We then discuss the implications of the findings and conclude in the final section.

2 Context

2.1 Nairobi's Matatus

Nairobi's transportation system was developed after independence in 1963 (Mutongi, 2017). Smallscale entrepreneurs responded to the growing demand for mobility by retrofitting old vehicles and transporting passengers from the suburbs to the urban center. The buses were labelled "matatus". meaning three in Kikuyu, in reference to the early ticket price in KES of a matatu ride. These private businesses were legalized in 1973, but remained largely unregulated until 2003 when the government passed the Michuki rules, requiring that buses install speed governors, safety belts, and ensure that all drivers exhibit valid licenses (Michuki, 2003). To date these regulations are rarely enforced. In 2010, the Ministry of Transport issued a new directive to further formalize the industry and eliminate the presence of gangs that were becoming increasingly active in the space. This required that all minibus owners form or join transport Savings and Credit Cooperatives (SACCOs) or Transport companies licensed to a particular route (McCormick et al., 2013). To this day industry newcomers must first register with a SACCO or Transport company before they can put their vehicle on the road. Transport companies are rare in Nairobi and manage buses on behalf of individual investors. SACCOs on the other hand leave the daily management of the vehicle to the owner, but facilitate centralized organizational activities including scheduling, resolving internal disputes between owners, ensuring compliance with the National Transport and Safety Authority (NTSA) regulations, and providing financial services to owners and drivers.

This informal network of buses constitutes the only dependable transit system in Nairobi, and the city comes to a near standstill on days when drivers strike. Rough estimates suggest that 15,000 to 20,000 buses currently circulate throughout the city, swerving on and off the road to collect passengers along their designated route. To this day the industry remains almost entirely locally owned: private entrepreneurs purchase 14 or 33 seat minibuses, and hire a driver to operate the vehicle along their SACCO's designated route. The presence of severe competition within a route explains the dangerous and reckless driving that prevails throughout the industry. According to the World Health Organization's Global Status Report on Road Safety, approximately 3,000-13,000 people die annually from traffic incidents where at least 30% of cases involve matatus (WHO, 2015). Conditions have not improved measurably in recent years. However, in an effort to combat negative stereotypes, matatu owners are increasingly investing in the comfort of their vehicle, the

aesthetic (colorful interior and exterior), the quality of the "experience" (helping passengers on and off the bus), and the perks (TV's) ((Reed, 2018)). There are no regulations placed on the aesthetic of the vehicle. Nevertheless, the more attractive and comfortable vehicles can charge up to twice the price of regular ones. Matatu fares vary between 0.5 and 1.5 USD for travel inside the city center, and between 1 to 5 USD for trips to the outskirts.

This public transportation industry is appealing to study for a number of reasons. First, it is representative of many other informal transit systems worldwide, including Tanzania's dala dala's, Haiti's tap tap's and India's rickshaws among others. Moreover, the sector is economically meaningfully in terms of the number of individuals it employs, and the amount of income it generates. In Kenya, estimates suggest that the industry employs over 500,000 people and contributes up to 5% of the country's GDP ((Kenya Roads Board, 2007)). Most importantly however, this context allows us to overcome major data constraints that have limited previous research in the space. Namely, we collect detailed information on the contract terms set by the employer and the actions of the employee (their choice of effort and lying). We also introduce exogenous variation into the costs of monitoring in order to observe changes to the contract.

2.2 Driver and owners

In this study we work exclusively with small firms that meet three basic criteria. First, the owners of these vehicles manage their matatu themselves, as opposed to hiring a third party manager. Second, the owners are not the primary drivers of the vehicle. These two conditions were designed to focus the research on the classical principal-agent relationship. Finally, we only worked with owners that had a single matatu at the time of recruitment. We chose single owner-driver pairs to remove any dynamics that arise from one driver reporting differently from another, sending competing signals for the owner to parse through. According to an exploratory survey we conducted in the pre-pilot phase of the experiment, approximately one quarter of matatu owners in the general population met these three criteria.

In this industry, owners have settled on a fixed rent contract with limited liability that is negotiated daily. Owners rent their vehicles to a driver every morning for an agreed upon "target price" (henceforth referred to as the 'target'). Unlike the taxi systems in many high-income countries, the driver is expected to deliver this amount at the *end* of the day once all the fares have been collected. This is primarily because drivers have limited capital and cannot afford to pay the amount up front. Drivers are the residual claimants in this contract and keep everything they earn above the target. The owner is not allowed to revise the terms of the contract and claim more revenue if the driver has had a good day. In the event that the driver cannot make the target, they are supposed to provide the total revenue they earned to the owner. In practice, drivers under-report total revenue to make sure they have some income left over. If they fail to make the target too many times, or

⁶Owners do not operate the vehicles themselves for two reasons. First, it allows them to pursue side-jobs that are more lucrative than being a driver. Second, driving is a tough job that individuals like to avoid if they have other options.

⁷If we allowed owners to posses two or three matatus, over 50% of matatu owners satisfied these conditions.

they are caught under-reporting too frequently they will be fired. Drivers can choose 1) the number of hours they work (effort), 2) their driving style (reckless driving that damages the vehicle), and 3) the amount of revenue they declare (under-reporting). Owners cannot directly observe these three actions by the driver, and must resort to costly interventions to check in on their drivers. This includes phone calls, dropping by the terminus of the route and staging someone at various stops to monitor whether the bus drives by.

This negotiation process is repeated daily over the phone (and occasionally in person). Formal documents are not signed because legal recourse is virtually non-existent. Typically owners and drivers have worked with each other for just over two years. The target price is set at approximately 3000 KES (30 USD) every day, and drivers make this target 44% of the time. On days when they do not report making above the target, they under-report revenue by approximately 700 KES. Drivers are typically on the road between 12-14 hours per day, and make approximately 10 trips to and from the city center.

This contract structure appears to be one of the only viable alternatives in this industry. A fixed wage payment is unattractive to most owners because drivers face incentives to undersupply effort when they cannot be monitored. The few SACCOs that have adopted this payment scheme have hired full time managers who supervise the drivers closely. Anecdotal evidence suggests that drivers also dislike this remuneration scheme because it eliminates the large windfall they receive on the best days. Next, a fixed rent contract is impossible to enforce because drivers are poor and hence the limited liability constraint always binds. This leaves the traditional sharecropping model or a fixed rent contract with limited liability. A sharecropping contract in this industry would have to take the form of a profit-sharing agreement where owners and drivers are each responsible for their share of the costs. However, mosts of the costs that the vehicle incurs are beyond the means of a matatu driver. A typical service fee is 2000 KES, which the driver simply cannot pay upfront. Similarly, in the extreme case that a matatu gets into an accident, drivers are known for running away from the scene. Moreover, a sharecropping model with unobserved output means that drivers can consistently underreport the amount they collect. The cost of under-reporting is low because drivers can easily hide undisclosed revenue.

A fixed rent contract with limited liability ensures that drivers face the incentive to supply effort when they make above the target. It also limits under-reporting to days when the driver does not report making the target. Note that due to the limited liability constraint, the supply of effort under this contract will be less than the first-best outcome because the driver does not supply optimal effort on days when they are not the residual claimant (they report below the target). This contract structure is prevalent in many informal transportation systems worldwide. It also characterizes relationships in agriculture where absentee landlords cannot supervise their tenants; in the service industry where employers cannot record the number of services provided by their employees; and in businesses where inventory is difficult to monitor.

2.3 Device (Hardware and Software)

Monitoring technologies are becoming widely available in many developing countries, including Kenya. The majority of long-range bus companies that travel between the country's main cities are equipped with tracking devices. Moreover, some banks in Nairobi recently announced that they would only issue loans for minibuses whose location could be tracked with a device. Despite their availability, most medium range buses and inner-city public transportation vehicles are not yet using them. When asked why, most vehicle owners citep the high cost of sophisticated tracking systems (approximately 600 dollars for the tracker and additional monthly installments for system access), or the lack of detailed information provided by the cheaper alternatives.

To fill this need, the research team created a new monitoring system for city buses that is considerably cheaper, more flexible and more powerful than traditional tracking devices. The physical tracking units were procured for 125\$ from a company in the United States (CalAmp). They feature GPS, internal back-up battery packs, 3-axis accelerometer for motion sense, tilt and impact detection. The device was designed to capture and transmit the information we required, including the 95th percentile and average forward/backward/lateral/vertical acceleration, as well as the 95th percentile and average forward/backward jerk. The device was also calibrated to generate alerts for every instance of vehicle speeding, over-acceleration, sharp braking and sharp turning. These safety alerts were calculated by an internal algorithm built into the CalAmp device with threshold parameters as inputs, using the full sequence of acceleration and speed data to identify unsafe driving actions. Further processing of the CalAmp system data on the server provided additional measures of interest including the total number of kilometers traveled that day, the total time the matatu was running, and a safety index (from aggregating the day's safety alerts). Finally, an API call was generated each time the owner used the app to request data from the server. These calls were recorded in the database and provided a measure of owners' usage of the app. In this way, we could track which types of information the owner found most valuable and how often the owner requested this information.

The data captured by the CalAmp device was transmitted to owners via a mobile application that was specifically designed to present information simply. The app (referred to as "Smart-Matatu") provided information in three ways (Figure 1). The first tab was a map of Nairobi and presented the real-time location of the vehicle. By entering a specific date and time interval into the phone, the app would display the exact routes traveled by the matatu over this time period. This first tab provided owners with a more accurate measure of driver effort because they could track the driver on the road. It also conveyed a more accurate measure of reckless driving because they could see if the driver was operating on bumpy routes that were damaging to the vehicle. The second tab displayed all the safety alerts that were captured by the device. The owner could click on the safety event to find out when and where it had occurred on the map. It is important to remember that these safety alerts are not necessarily correlated with reckless driving behaviors such as off-route driving. The final tab conveyed a summary of the driver's productivity and safety. The productivity section of this page listed the total mileage covered, and the duration the ignition was

turned on that day. This could be used by owners to estimate total revenue more precisely. The safety section of this page provided the owner with an overview of the number of safety violations that occurred that day, as well as the driver's daily safety rating relative to all other drivers on the road that day (where a thumbs up appeared for scores of 60% and above, a sideways thumb for scores between 40% and 60%, and a thumbs down for scores of 40% and below).

3 Experimental Design

3.1 Sample Recruitment

We conducted an extensive recruitment drive in late 2015 by contacting 61 SACCOs that were operating along various routes across the city. We organized several large meetings with matatus owners in each SACCO, presenting the study's goals and methodology. We also continuously relied on referrals from the owners we were interacting with to increase awareness about the project. All owners were informed at the time of recruitment that a monitoring device would be placed in their vehicle free of charge, and they would be required to provide daily information about their business operations. We also mentioned that a random subset of owners would be selected to receive information from the tracker via a smartphone app for a six month time period, while others would have to wait 6 months before gaining access to the information for a shorter 2 month period. Recruitment took 4 months in order to secure enough participants. Owners who expressed interest in the study during the recruitment drive were contacted again over the phone to confirm their willingness to participate in the experiment, and to check that they met the three study requirements (owners had to own a single matatu, which they rented to a driver, and manage the firm's operations themselves).

3.2 Installations

The first installation took place in November 2016, and continued until April 2017. The field team, managed by EchoMobile, was able to fit approximately 15 matatus per week with a device (Figure 2). The team scheduled a time to meet each owner individually at a location of their choosing. The owner was compensated for the time their vehicle spent off-road to perform the installation of the device with a one-time payment of 5000 KES (50 USD) and a new Android phone (worth approximately 80 USD) to ensure they could access the SmartMatatu app. The installation process was rather complex, requiring a team of 3 staff (an enumerator, a field manager, and an engineer). While the mechanic worked on fitting the device in the matatu, the field manager took the owner aside to re-explain the purpose of the research project and the tracking device's functionality. For owners in the treatment group, the field manager conducted an additional training on the app. At the same time, the enumerator administered the baseline survey to the driver in a separate location, outside of the owner's earshot, so that the driver felt comfortable answering the questions honestly. Once the field manager finished the training with the owner, and the enumerator finished administering the survey to the driver, they switched. The field manager then took the driver aside

to explain that they would receive a daily SMS to elicit information about the day's operations and to emphasize that all of the data they shared would remain confidential. Meanwhile, the enumerator conducted a 20-minute baseline survey with the owner. This whole installation process took approximately 1 hour to complete. The field manager shared his contact information with the owner and the driver so they could contact him with any further questions they had.

3.3 Treatment Assignment

The first treatment arm is referred to as the "information treatment". Owners in our sample were randomly allocated to a treatment and a control group. Owners in the treatment group were provided with free access to the data produced by the monitoring device immediately after installation. Owners in the control group were informed that they would receive the same access six months after the device was installed. During the device installations our field manager spent an additional 30 minutes with treatment owners explaining the types of data that would be visible on the SmartMatatu app. A small survey was administered with the owners at the end of their training to make sure they knew how to find all the information contained in the app. We informed treatment and control drivers that a tracker would be placed in their vehicle. We did not mention, however, whether the information would be transferred to the owner. This meant that any subsequent changes we observed in driver behavior could only come from owners using the tracker data, and not from receiving different information from enumerators during the installation.

Four months after the information treatment was launched we introduced a second treatment arm, referred to as the "safety" treatment. We selected half of the treatment drivers and half of the control drivers and offered them cash incentives to drive safely. This arm was designed to simulate the role of a functioning regulatory system and monetize the tradeoff between revenue and safety that drivers face. The cash incentive drivers were again randomly split into two groups: a one-month treatment group and a two-month treatment group. This was done so we could study whether any changes in driving behavior that might be induced by the incentives would persist after they were removed. The specific incentive amount they received was determined by a safety rating, calculated daily for each driver in the following way. We analyzed two weeks of data for each driver (dropping days with less than 30km), tracking 1) the number of alerts of each type k (speeding, heavy braking, sharp turning and over-acceleration), and 2) the number of hours worked. For each driver, day and alert type we computed the rate of violations by dividing the number of alerts of type k on a given day t for driver i by the number of hours worked a given day t for driver i. For each driver i, we then construct a distribution of these rates for each alert type k and found the percentile that day's alert rate falls in. We then calculated the weighted average percentile for driver i on day t, by adding the alert rates for each type, applying weights of 1/3 for over-speeding and breaking, and 1/6 for over-acceleration and turning. The average lies between 1 and 100, and for each driver on each day, we then assess the cutoff below which they fall and disburse their incentives accordingly.

4 Data Collection

We collected data from three different sources. The first data set is a panel of daily responses from owners and drivers which we gathered through the app and SMS surveys, respectively. Next the enumerators conducted 8 monthly surveys, beginning with the baseline, followed by 6 monthly surveys and wrapping up with the endline. Finally the GPS tracker collected a wealth of data that we use to measure safety violations committed throughout the day, and driving behavior.

4.1 Non-system application variables

The SmartMatatu app was also designed to collect information from owners. Collecting accurate data can be very challenging in these settings, and this system was created to improve the quality of the data we received. Owners in the study were reminded daily via a push notification to report on that day's business activities through a form located on the app. They were asked to submit data on: the "target" amount assigned to their driver at the beginning of the day; the amount the driver delivered to the owner; an overall satisfaction score for their driver's performance (bad, neutral, good); and whether the driver was fired/quit that day. Once the report was successfully submitted, owners received 40 KES via M-Pesa. We collected similar information from drivers through SMS surveys (because the drivers were not provided with smartphones). At the end of every work day around 10pm, drivers would receive a text message asking whether they were ready to respond to the survey. Once they agreed, individual text messages were sent to the driver asking for: the total revenue the matatu collected from fares that day; any repair costs incurred, the amount they spent on fuel; and their "take home salary" (their residual income after expenses and paying the owner). Once the driver responded to all the questions, they were sent 40 KES via M-Pesa to incentivize consistent reporting.

We developed a set of processes for checking and validating the daily data we receive from owners and drivers. Echo Mobile wrote code to check for anomalies including outliers and entries that did not make sense and/or suggested the owner/driver may not have understood how to answer the question. A team of enumerators would then follow up with owners and drivers over the phone about each one of these entries. In cases where owners and drivers were able to justify their responses, the enumerators would record their justifications in an excel spreadsheet. The necessary changes were made when the data needed to be corrected.

4.2 Monthly Surveys

We conducted eight rounds of surveys. We first administered the baseline surveys during the tracker installation. The *owner* baseline survey collected detailed information regarding basic demographics, employment history, features of the matatu, and their relationship with the current driver. Similarly the *driver* baseline asked about driver demographics, experience as a driver, unemployment spells, and their relationship with the current owner. For both owners and drivers we measured cognitive ability through raven's matrices. We also used games to gauge drivers' risk

aversion and driver/owner propensity to trust one another. To measure risk we asked respondents whether they would prefer to receive 500 KES for certain or play a lottery to win 1500 KES. The game was repeated multiple times, with increasingly favorable lottery odds. The trust game on the other hand presented owners with 500 KES and asked them to select a certain amount to be placed back in an envelope. They were informed that this amount would be tripled and delivered to a random matatu driver who was then going to decide how much to keep for himself and how much to return to the owner. The amount they chose to place in the envelope was recorded in the survey. When playing the game with drivers, we first presented them with an envelope containing 900 KES. This amount was standardized across all drivers to ensure they faced the same choice. The drivers were informed about the owner's decision and how this amount was then tripled. The drivers were asked to return however much they wanted to the same owner.

We proceeded with 5 monthly follow up surveys. The monthly surveys were administered with three purposes in mind. First, they provided an opportunity for enumerators to follow up regularly with matatu owners and drivers and address any questions they might have about the device. Second, they were used to remind both parties to continue submitting the daily reports in the SmartMatatu app. Finally, they were designed to track changes in the owner-driver relationship (how much monitoring had taken place, how satisfied the owner and driver were with the others' management/driving respectively), and any large expenses the business may have incurred that month. As owners and drivers reached the 6-month mark, we conducted an endline survey to measure changes in key outcomes, and to assess the impact of the information treatment and the cash incentives on owners and drivers respectively.

4.3 Tracking data

The CalAmp tracking device transmitted high frequency data on forward/backward/lateral/vertical acceleration, jerk, location and a timestamp. We use these raw measures of acceleration to investigate changes in driver behavior (where we aggregate the 30-second panel to the day level). We also use the GPS data to calculate how far each vehicle is from the route they are licensed to. This gives us a measure of how far the driver is deviating from the actual route. Finally, figure 3 depicts the number of times a vehicle on route 126 passes through a particular lon-lan cell. The first panel clearly shows what the route should be, and the second panel overlays the designated route to confirm. The figure confirms that off-route driving is common practice.

The tracker subsequently fed the raw data into an algorithm that computed the number of safety events that occurred in a 30 second time frame. Thresholds were calibrated for the Kenyan roads to avoid capturing an unreasonable number of safety violations and losing credibility among owners. These events included instances of speeding, over-acceleration, sharp braking, and sharp turns. The data was then further aggregated on the backend to produce daily reports on the number of safety violations, which is what we use for our analysis.

5 A Principal-Agent model with unobserved output

The purpose of the model is to generate key predictions about how the new monitoring device affects the principal-agent relationship with unobservable output and unobservable effort. The owner chooses the target, while the driver chooses the amount of effort and reckless driving they engage in. Reckless (or risky) driving refers to the set of behaviors that damage the company asset (which are not necessarily correlated with the unsafe driving metrics we will discuss in the results section). The driver also decides when they will under-report revenue, and by how much. This results in five key parameters, and we derive comparative statics for each one. For simplicity we assume that both owners and drivers are risk-neutral.

5.1 Status Quo

The model is comprised of 4 steps that correspond to the owner-driver daily interaction. First, the owner sets the target that the driver is expected to deliver by the end of the day. Second, the driver chooses how much effort and reckless driving to engage in. Together with the day's random events, total revenue is produced. Third, the driver chooses how much revenue to report to the owner. Finally the owner decides whether or not to punish based on his beliefs about what revenue should have been by the end of the day. We solve the model in 4 steps, via backward induction.

Step 1: Owner's beliefs and punishment

The owner (principal) sets a target amount (T) at the beginning of the day, which they expect the driver (agent) to deliver by the end of the day. The driver earns revenue (q) from passenger fares, and decides how much to report to the owner (\tilde{q}) . We define \hat{q} to be the owner's belief about true revenue (q).

$$\begin{split} \hat{q} &= q - \sigma \\ \sigma \sim U\left(-\frac{1}{\alpha}, \frac{1}{\alpha}\right) & f(\hat{q}) &= \frac{1}{\hat{q} + \frac{1}{\alpha} - \left(\hat{q} - \frac{1}{\alpha}\right)} = \frac{\alpha}{2} \end{split}$$

Where α is the precision of the owners signal about true revenue. Any monitoring technology we introduce will increase the precision of the owner's signal, which gives the driver less leeway to significantly under-report revenue on a particular day.

On days when the driver reports making the target, the owner receives the target amount and does not punish the driver under any circumstances. On days when the driver does not report making the target, the owner will punish them if the reported revenue comes in below the range the owner expected revenue to fall in. The actual punishment applied is some function of the difference between the lowest value the owner expected revenue to be and the reported amount (owners are less upset on days where the driver reports below the target and they know for a fact that conditions were difficult). We assume for simplicity that this function is linear. It is worth mentioning that this punishment is not monetary (it would be impossible to enforce such a measure) but rather a

verbal reprimand which can lead to the termination of the contract if incurred frequently.

$$\begin{split} E[punishment] &= E\left[\left(\hat{q} - \frac{1}{\alpha}\right) - \tilde{q} \mid \hat{q} - \frac{1}{\alpha} > \tilde{q}\right] \cdot \Pr\left(\hat{q} - \frac{1}{\alpha} > \tilde{q}\right) \\ &= \int_{\tilde{q} + \frac{1}{\alpha}}^{q + \frac{1}{\alpha}} \left(\hat{q} - \tilde{q} - \frac{1}{\alpha}\right) \cdot f(\hat{q}) d\hat{q} \\ &= \frac{\alpha}{4} \left(q - \tilde{q}\right)^2 \end{split}$$

Step 2: Solve the agent's optimal shading amount

Case 1: The driver chooses to report above the target $(\tilde{q} > T)$

When the agent chooses to report above the target, they do not face any incentive to lie because they keep everything they earn above the target and the owner cannot renegotiate the terms of the contract. Therefore they report truthfully $\tilde{q} = q$ and their utility is

$$U^D = \overbrace{q - T}^{salary} - \overbrace{\beta r}^{damages}$$

where β is the probability the owner detects risky driving that damages the vehicle (note that without a monitoring technology β is close to zero because the owner has a hard time detecting behavior that damages the vehicle).

Case 2: The driver chooses to report below the target $(\tilde{q} < T)$

When the agent chooses to report below the target, they face an incentive to lie in order to increase their take-home pay. Indeed, on days when the driver truly fails to make the target q < T, underreporting revenue means they get to walk away with some money rather than handing it all over to the owner as stipulated by the contract. Even when the amount of revenue is slightly above the target q > T, the driver faces some incentive to lie in order to walk away with slightly more income than what they otherwise would if they reported truthfully and had to hand over the entire target amount. On these set of days where the driver decides to report below the target, the owner will punish them based on what they believe revenue should have been. The driver then needs to choose the amount of revenue to report (\tilde{q}) to maximize their utility:

$$\max_{\tilde{q}} \quad U^D = \overbrace{(q - \tilde{q})}^{salary} - \underbrace{\frac{\alpha}{4}(q - \tilde{q})^2}_{punishment} - \overbrace{\beta r}^{damages}$$

Solving for \tilde{q} yields:

$$\tilde{q} = q - \frac{2}{\alpha}$$

Where

$$\frac{\partial \tilde{q}}{\partial \alpha} = \frac{2}{\alpha^2} > 0$$

Which says that the optimal amount for the driver to under-report (shade) is a function of the owner's signal. More specifically, it is optimal for the driver to under-report by a constant amount $(\frac{2}{\alpha})$. Figure 4 confirms this behavior in the data. The graph summarizes values of shading at unique/binned values of net revenue above the target. We see that divers continuously shade approximately 700 KES (7 USD) until the net revenue they generate exceeds the target by approximately 500-1000 KES.⁸

Step 3: Switch point

Next we need to determine the point at which the driver is in different between reporting above the target (and telling the truth) and reporting below the target (and under-reporting). When the driver tells the truth i.e $\tilde{q}=q$, they get utility:

$$(q-T-\beta r)$$

When they lie "optimally" i.e $\tilde{q} = q - \frac{2}{\alpha}$, they get utility:

$$(q - \tilde{q}) - \frac{\alpha}{4}(q - \tilde{q})^2 - \beta r$$

Setting the two utilities equal and solving:

$$q - T - \beta r = (q - \tilde{q}) - \frac{\alpha}{4}(q - \tilde{q})^2 - \beta r$$
$$q^* = T + \frac{1}{\alpha}$$

Where

$$\frac{\partial q^*}{\partial \alpha} = -\frac{1}{\alpha^2} < 0$$
$$\frac{\partial q^*}{\partial T} = 1 > 0$$

Which says that the revenue required to truthfully report and make the target (q^*) is a function of the owner's signal and the target.

 $^{^8}$ To get an accurate measure of shading we want to know the share of joint revenue that the driver withholds. In other words we need to know the income that the owner took home and and the salary of the driver. We therefore use net revenue above target on the x-axis, defined as owner income + driver salary - target

Step 4: Driver's optimal choice of effort

The driver chooses two actions, effort (e) and risk (r), which affect the probability distribution of revenue. For simplicity we assume that an increase in effort shifts the distribution of output to the right, and an increase in risk reduces the variance of revenue while keeping the mean constant (this is similar to (Ghatak and Pandey, 2000)):

$$q = e + r \cdot \varepsilon$$

The driver chooses effort and risk to maximize his utility

$$\max_{e,r} \underbrace{E\left[(q-T-\beta r) \mid q \geq q^*\right] \cdot Pr(q \geq q^*)}_{Truth} + \underbrace{E\left[(q-\tilde{q}) - \frac{\alpha}{4}(q-\tilde{q})^2 - \beta r \mid q < q^*\right] \cdot Pr(q < q^*)}_{Shade} - h(e,r)$$

Which yields the following F.O.C with respect to e and r, respectively (all the derivations in the paper can be found in the Appendix):

$$1 - F_{\varepsilon} \left(\frac{q^* - e}{r} \right) - h'_{e} = 0$$
$$\int_{\frac{q^* - e}{r}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon - h'_{r} - 2\beta = 0$$

Where

$$\begin{split} \frac{\partial r}{\partial \alpha} &< 0 &\& \frac{\partial e}{\partial \alpha} > 0 \\ \frac{\partial r}{\partial T} &> 0 &\& \frac{\partial e}{\partial T} < 0 \\ \frac{\partial r}{\partial \beta} &< 0 &\& \frac{\partial e}{\partial \beta} > 0 \end{split}$$

Which says that the driver's choice of effort and risk is a function of the owner's signal, the target, and the probability the owner detects risky driving.

Step 5: Owner's choice of the target

Constrained case

The owner chooses T to maximize his utility:

$$\begin{split} & \max_{T} \quad T \cdot Pr(q \geq q^{*}) + E[\tilde{q} \mid q < q^{*}] \cdot Pr(q < q^{*}) - \gamma(r) \qquad \text{s.t} \\ & E\left[(q - T - \beta r) | q \geq q^{*}\right] \cdot Pr(q \geq q^{*}) + E\left[(q - \tilde{q}) - \frac{\alpha}{4}(q - \tilde{q})^{2} - \beta r \mid q < q^{*}\right] \cdot Pr(q < q^{*}) - h(e^{*}, r^{*}) > 0 \end{split}$$

Which yields the following F.O.C with respect to T and λ , respectively:

$$\frac{\partial}{\partial T} = 1 - F_{\varepsilon}(\cdot) + \frac{\partial e}{\partial T} F_{\varepsilon}(\cdot) + \frac{\partial r}{\partial T} \int_{0}^{\frac{q^{*} - e^{*}}{r^{*}}} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon - \frac{\partial}{\partial T} \left(\frac{q^{*} - e^{*}}{r^{*}}\right) \left(\frac{1}{\alpha}\right) f_{\varepsilon}(\cdot) - \gamma'(r) \frac{\partial r}{\partial T} + \lambda \left[-(1 - F_{\varepsilon}(\cdot)) + \frac{\partial e}{\partial T} (1 - F_{\varepsilon}(\cdot)) + \frac{\partial r}{\partial T} \int_{\frac{q^{*} - e^{*}}{r^{*}}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon + \frac{\partial r}{\partial T} (-\beta) - h' \left(\frac{\partial e}{\partial T} + \frac{\partial r}{\partial T}\right) \right] = 0$$

$$\frac{\partial}{\partial \lambda} = \int_{\frac{q^{*} - e^{*}}{r^{*}}}^{\infty} \left(e^{*} + r^{*}\varepsilon - T - \beta r^{*}\right) f_{\varepsilon}(\varepsilon) d\varepsilon + \int_{0}^{\frac{q^{*} - e^{*}}{r^{*}}} \left(\frac{1}{\alpha} - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon - h(e^{*}, r^{*}) = 0$$

Unconstrained case

The owner chooses T to maximize his utility:

$$\max_{T} \qquad T \cdot Pr(q \ge q^*) + E[\tilde{q} \mid q < q^*] \cdot Pr(q < q^*) - \gamma(r)$$

Which yields the following F.O.C with respect to T

$$(1 - F_{\varepsilon}(\cdot)) + \frac{\partial e}{\partial T} F_{\varepsilon}(\cdot) + \frac{\partial r}{\partial T} \left(\int_{0}^{\frac{q^* - e^*}{r^*}} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon \right) - \frac{1}{\alpha} f_{\varepsilon}(\cdot) \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) - \gamma'(r) \frac{\partial r}{\partial T} dr$$

5.2 Monitoring Technology

Introducing the monitoring technology increases α and β . In other words, the precision of the owner's signal about output (α) , and the probability of detecting risky driving (β) , increase. In what follows we consider how the owner changes the terms of the contract, and how this subsequently affects the driver's choices of effort, risk and shading. We assume throughout that the driver's participation constraint binds.⁹ As α and β increase:

$$\frac{\partial T}{\partial \alpha} < 0 \quad and \quad \frac{\partial T}{\partial \beta} < 0$$

This says that as the precision of the owner's signal (α) increases, the owner will reduce the target. We know that the driver's utility will fall as α increases. Because the constraint binds, the owner needs to reduce the target to ensure the driver makes their reservation wage. The same reasoning explains why the owner reduces the target as the probability of detecting risky driving (β) increases.

⁹Anecdotally we know that there are a lot of drivers on the market and we think it reasonable to assume that they would have been bargained down to their constraint. Switching costs are not trivial and the owner would prefer less turnover all else equal.

Proposition 2 (Risk - Damaging Driving)

$$\frac{dr}{d\alpha} = \frac{\partial r}{\partial \alpha} + \frac{\partial r}{\partial T} \frac{\partial T}{\partial \alpha} < 0$$

$$\frac{dr}{d\beta} = \underbrace{\frac{\partial r}{\partial \beta}}_{+} + \underbrace{\frac{\partial r}{\partial T}}_{+} \frac{\partial T}{\partial \beta} < 0$$

This says that risk will unambiguously decrease when α increases. There are two effects at work. As α increases, the driver is going to have to truthfully report more often $(q^* \text{ falls})$, which increases the probability they are the residual claimant and must bear the cost of a bad revenue day. Being exposed to greater downside risk makes risky behavior less attractive. Similarly, as the owner reduces the target in response to a higher α , $(q^* \text{ falls})$, and the driver will take on less risk. Next, the driver's immediate response to an increase in probability of detection (β) is to reduce risk. Moreover as the owner reduces the target in response to higher β , the driver will further reduce the amount of risk they take.

Proposition 3 (Effort)

$$\frac{de}{d\alpha} = \underbrace{\frac{\partial e}{\partial \alpha}}_{+} + \underbrace{\frac{\partial e}{\partial T}}_{-} \underbrace{\frac{\partial T}{\partial \alpha}}_{-} > 0$$

$$\frac{de}{d\beta} = \underbrace{\frac{\partial e}{\partial \beta}}_{+} + \underbrace{\frac{\partial e}{\partial T}}_{-} \underbrace{\frac{\partial T}{\partial \beta}}_{-} > 0$$

This says that effort will unambiguously increase when α increases. Yet again, there are both direct and indirect effects driving this result. As the precision of the owner's signal, α , increases, the driver will supply more effort $\frac{\partial e}{\partial \alpha} > 0$. Indeed, for every level of output below q^* , the driver can no longer shade as much as they used to because the owner has a more precise signal of what output should be. Holding all else constant, their revenue will decrease if they do not respond by increasing effort and generating more revenue. Next, we know that the owner responds to an increase in α by lowering the target. As the target decreases the driver is more likely to make it, which increases the returns to effort and incentivizes the driver to work more. Turning next to the effects of detecting risky driving (β) more heavily: we know from above that the driver will have to reduce the amount of risk they take, which reduces the probability of large windfall days. The driver has to compensate by investing more effort.

Proposition 4 (Switch Point)

$$\frac{dq^*}{d\alpha} = \overbrace{\frac{\partial q^*}{\partial \alpha}}^{-} + \overbrace{\frac{\partial q}{\partial T}}^{+} \overbrace{\frac{\partial T}{\partial \alpha}}^{-} < 0$$

As the owner's signal becomes more precise, the revenue required to truthfully report and make the target q^* will decrease. Similarly, as the target decreases, the revenue required to truthfully report and make the target q^* will decrease. Together these two factors drive the reduction in q^* we anticipate with the introduction of the monitoring technology.

Proposition 5 (Under-reporting)

$$\frac{d\tilde{q}}{d\alpha} = \frac{2}{\alpha^2} > 0$$

Which says that the optimal amount for the driver to report (\tilde{q}) increases with the precision of owners' signal (α) . It follows mechanically that the amount they under-report (shade) will fall.

6 Results

6.1 Baseline Characteristics

We work exclusively with matatu owners with one vehicle, which they do not operate themselves. They are approximately 40 years of age, and have completed 11 years of education. These small-scale entrepreneurs have spent an average of 8 years in the matatu industry, owning a vehicle for the past 4 years. While it is possible to have a salaried job and manage a matatu at the same time, only 20% of our sample juggle these two responsibilities. Typically owners have worked with their current driver for the past 2 years. Drivers have very similar profiles - which we expect - because many owners were previously driving matatus themselves. They are a few years younger (35 years old on average), with slightly lower levels of education (8 years on average). They have worked in the industry for over a decade, driving a vehicle for the past 7-8 years. They have worked with 5 different owners, averaging 1.5 years with each one. Both driver and owner characteristics are balanced across treatment and control groups (Table 1 and Table 2).

6.2 Information Treatment Arm

To study the treatment effect of information on contracts, productivity (which includes effort, risk, revenue, costs and profits) and safety, over the 6 month time frame we run the following regression model:

$$y_{ird} = \alpha_d + \tau_r + \sum_{m=1}^{6} D_{im} \beta_m + X_i \gamma + \varepsilon_{ird}$$

where y_{ird} is a daily contract/productivity/safety outcome for owner i on route r, on day since installation d; α_d is a day fixed effect; τ_r is an assigned route fixed effect; D_{im} is a treatment indicator equal to 1 if the owner is in the information group in month m (which allows us to examine the treatment effect as it evolves over the six months of the study), X_i is a set of firm-

level baseline specific controls included for precision, and ε_{irmd} is an error term.¹⁰ We cluster our standard errors at the firm level. Note that the study offered the information to control owners in months 7 and 8 (as compensation for participating in the study). As a result all the regressions only include data before month 7.

We also have an endline survey that asked owners about their use of the device, perceptions of drivers' performance, their monitoring strategies, and their firm's size. To study the impact of our device on these outcomes we run the following regression model:

$$y_{ir} = \alpha_d + \tau_r + D_i \beta + X_i \gamma + \varepsilon_{ir}$$

where y_{ir} is an endline outcome for owner i on route r; τ_r is an assigned route fixed effect; D_i is a treatment indicator equal to 1 if the owner is in the information group; X_i is a set of firm-level baseline controls included for precision, and ε_{ir} is an error term.

6.3 Usage

First, we monitor owners usage of the device. We do so by tracking the API calls that are generated every time the owner logs into the app and requests different pieces of information (including historical location, up-to-date summary information, where the safety violations occurred on the map). Figure 5 calculates the share of owners that made at least one API call during the week. We find high rates of take-up. In the early months of the study approximately 80% of owners are checking the app at least once a week. This share decreases but stabilizes at about 70% as the study progresses. A large share of owners are also using the app daily. In the first few months, 60% of owners check the app once a day, and 40% continue their daily usage after 6 months. This suggests that owners are actively engaging with the device throughout the study.

We also check whether owners are internalizing the information we provided. At endline we asked owners to state the revenue earned, the number of kilometers driven, the fuel costs, and the extent of off-road driving on the most recent day their vehicle was active. Owners had the option of answering "don't know". We find that owners in the treatment group are 27 percentage points more likely to know about the number of kilometers driven and 45 percentage points more likely to know about the the instances of off-route driving (Table 3 Columns 1 and 2). We do not find any differences between the treatment and control groups regarding knowledge of revenue, which is not captured by the device (Column 3). As a final test, we also ask owners to rate how challenging it is to monitor their employees on a scale from 1 (not hard) to 5 (very hard). Having access to the information reduces the reported difficulty level by just over 2 points. In other words, control owners maintain that monitoring is hard while treatment owners reveal that it is easy (Table 4). We do not find any significant changes in traditional monitoring behaviors (checking-in with the driver over the phone, at the stage, through a third party).

¹⁰Controls include the matatu's age and number of features, as well as owner's age, education, gender, tenure in the industry, their raven score and the number of other jobs they have.

Finally, we investigate whether there are any changes in how the owners and driver interact. We asked drivers to report the frequency with which they were contacted and criticized by the owner that month. Formal reprimands are not frequent but they are used by owners to signal their displeasure with the driver's behavior. Figure 6 suggests that the number of reprimands is marginally higher in the treatment group at the beginning of the study period. This is consistent with the idea that owners use the information to correct behavior early on. The frequency increases by approximately 20-30 % (off of a control mean of 0.5) in months 1-4 before decreasing significantly in month 6. We also investigate whether owners take more extreme actions and fire their drivers more frequently. While the trend in Figure 6 suggests that the number of firings increased in the second month of the study and decreased thereafter, this result should be interpreted carefully because there are so few firings in our data (17 in total).

6.3.1 Contracts

We first investigate whether access to the tracking information changes the *terms* of the contract. While the intervention could also have changed the *type* of contract they offered their drivers (fixed wage or sharecropping), extensive interviews with owners suggested this was unlikely to occur. The fixed wage contract is unpopular among owners and drivers, and the sharecropping model is difficult to implement when limited liability constraints bind, and revenue is unobserved/can be easily withheld by the drivers. Moreover social norms are engrained in this industry, and a change of this magnitude would be unexpected in a 6 month time frame. We further confirmed in our endline survey that every owner maintained the same type of contract.

As detailed in the model, owners can use the information from the device to change the target they set for drivers (Proposition 1). Absent the technology, the target for 14 seater buses is usually set at 3000 KES. Discussions with owners confirm this is an industry standard that only fluctuates with good reason (they know that demand will be high or low that day because the weather or road conditions have changed). Charging much more would alienate drivers, and charging any less would cut into firm revenues. Figure 7 depicts the estimated treatment effect on the owners' daily target across the 6 months of the study. There are no significant changes in the first month, likely because owners were still learning how to use the app and experiment with ways to improve their business operations. In subsequent months, however, we see the target steadily declining. By month 6, the target amount is 135 KES below the control group, representing a 4.5% decrease (0.2 standard deviation). While the result is not statistically significant (likely because we are underpowered), the downward trend is clear. This steady reduction suggests that the information allows managers to re-optimize the terms of their employees' contracts. Taking this result back to the model, it suggests that the drivers are operating at their participation constraint. When the constraint binds, the owner needs to decrease the target in order to compensate the driver for their lost information rents. Lowering the target also reduces owners' revenue on days where the driver makes the target. As a result it is only profitable for the owner to do so if they are compensated in other ways, namely with a higher share of revenue on days when the driver does not make the target, fewer damages to the vehicle, or an increase in the frequency with which drivers makes the target. We turn to these results next.

6.3.2 Productivity

We consider three measures of productivity, which correspond to the choices that drivers make throughout the day. This includes how recklessly/damaging they will drive, how much effort to supply, and the amount of revenue they disclose to the owner (which is either the target amount, or some amount below).

1. Reckless/Damaging driving: We hypothesize that owners prefer less reckless (damaging) driving than what the drivers would optimally choose. With the technology, owners can observe driving behavior more accurately, and the probability they detect reckless driving increases. This reduces drivers' incentive to drive recklessly, and damages to the vehicle should be lower. Similarly, as the precision of the owner's signal about revenue improves, and they decrease the target, the driver will be exposed to greater downside risk on bad revenue days. This should further reduce their incentive to drive recklessly (Proposition 2). Figure 8 confirms this hypothesis in the data. We see damages substantially decrease throughout the entire 6 month period. In month 2 daily repair costs for treatment owners are reduced by 100 KES, and continue falling until month 6 where they are 250 KES less than what control owners incur on average (this represents a 50% decrease in daily repair costs). These repair costs represent a major business expense for owners, which makes the impact of the monitoring technology significant.

We want to confirm that this result stems from less reckless driving behavior. One of the greatest sources of reckless driving is off-route driving. Drivers often take shortcuts on bumpy roads that are notoriously damaging to matatus. These shortcuts are appealing to the driver because they help them travel to the city center more quickly, and avoid traffic jams where they sit ideally without picking up any passengers. Typically owners cannot observe off-route driving and drivers cannot be expected to pay for vehicle repairs. This means that reckless driving along these alternate routes is costless to drivers. When owners have access to the monitoring technology, however, they can inform drivers about how to take better care of the vehicle, and mandate that they stay on their designated routes. To investigate this hypothesis, we compute the distance between each GPS point recorded by the device, and their designated licensed route that vehicle is supposed to be on. Figure 9 demonstrates that treatment drivers are on average 400 meters closer to the designated route than control owners. To confirm that the reduction in off-route driving is responsible for fewer damages, we investigate whether the distributions of lateral and vertical acceleration differ across treatment and control groups. Lateral acceleration measures tilting from side to side, while vertical acceleration captures movement upwards and downwards. We might expect these measures

 $^{^{11}}$ As detailed in the model, the switch point q^* shifts down. This means that there are days when the driver used to shade, and they now truthfully report. As the residual claimant on these days, they bear the cost of a bad revenue day (which are higher when they make more risky maneuvers on the road).

to be different if drivers operate less along these alternative routes that are less frequently paved. Taking fewer bumpy roads that jostle the vehicle should be visible in the acceleration data. We find suggestive evidence that driving behavior has changed. The distribution of lateral acceleration in the treatment group tightens around 0 (less tilting - Figure 10). Similarly, the distribution of vertical acceleration has more mass around gravity (normal driving) for the treatment group. We can reject equality of these distributions across treatment and control by applying a K-S test, which returns a p-value of 0.000 for both measures of acceleration.

It is also important to rule out any alternative explanations for these effects on repair costs. Specifically, it could be the case that drivers tend to inflate repair-costs, and the device reduces their incentive to do so because they are more likely to be caught in the lie. This cannot be the case for larger repairs, however, because they are incurred by the owner directly and/or will be validated with the mechanic. We therefore create an indicator for whether the repair costs exceed 1000 KES (80th percentile). The second panel in Figure 8 demonstrates that the probability of incurring a large repair cost decreases significantly (7-8 percentage points). This implies that the decrease in the repair costs that we observe cannot be entirely driven by driver inflation. Drivers are also changing how they drive as the result of the technology.

- 2. Effort: Next, we proxy driver effort by the number of hours the tracking device is on (the device powers on and off with the matatu). When the device is installed in the matatu, drivers know that owners have a more precise signal of output, which means they are more likely to get caught if they under-report heavily. This encourages drivers to under-report by less, which reduces their take-home pay. As the model demonstrates, this creates an incentive for drivers to invest more effort throughout the day so they can increase total revenue and ensure they maintain similar compensation. In parallel, owners have lowered the target which means that becoming the residual claimant is more achievable. Finally, the model predicts that drivers will compensate for the reduction in risky driving by investing more effort. For all of these reasons we expect effort levels to rise (Proposition 3). This prediction is borne out in our data: Figure 11 demonstrates the upward trend in effort that we anticipated. The number of hours the tracking device is on increases by 0.9 hours in month 2 and rises steadily until the end of the study. By month 6, effort levels increase by 1.4 hours in the treatment group. This represents a 12% increase in drivers' labor supply. With more hours on the road, we also see the number of kilometers increase by 12 kilometers per day on average.
- 3. Reporting Behavior: Once the driver chooses how much effort and risk to invest, they need to decide whether or not to truthfully report. The model predicts that the optimal switch point for truthfully reporting (q^*) shifts down because 1) the owner's signal of true revenue becomes more precise, and 2) the owner has lowered the target (Proposition 4). Testing this proposition is difficult because we do not have a direct measure of q^* , we only observe whether drivers make the target or not. However, a lower q^* implies that drivers should make the target more often, primarily on days when revenue is close to q^* to begin with. To investigate this prediction in the data, we first

apply our standard regression specification to determine whether we see a significant change in the probability of making the target. Figure 12 suggests that from month 3 onwards, the rate at which drivers make the target increases by 11 percentage points off of a base of 44 percent (significant in month 3 only). It is not altogether surprising that the result is slightly weaker because the analysis considers the full range of revenue rather than focusing on days when drivers are close to q^* to begin with (i.e. close to making the target). This is where the model predicts we should see these effects. To investigate this further we calculate the average revenue above target on a route-month in the control group to get a sense of the usual revenue above target generated for a day. 12 We then compute drivers' daily reported revenue above target and subtract the average expected amount. This is akin to including route fixed effects, because we know that a certain level of revenue above target will be acceptable on certain routes but not on others. We are left with a measure of daily deviation from expected revenue above target, which we plot in the second panel of Figure 12. The revenue above target measure has an approximate mean of 4,000 KES. As such, -2000 KES on the graph implies that drivers only have 2,000 KES in revenue to cover their salary and their costs for that day. This results in a take-home pay of 500 to 1000 schillings, which is right where we expect q^* to be (from plotting the amount drivers under-report in Figure 4). Figure 12 demonstrates that the probability of making the target increases significantly at this point, which is exactly what we would expect. This represents a meaningful increase in "compliance" with the terms of the contract.

Finally, we anticipate that drivers' reporting behavior (\tilde{q}) will change as the monitoring devices are introduced. According to the model we should see drivers under-reporting below some optimum q^* , at which point they will start truthfully reporting and providing the owner with the target amount. Below this optimum, the model predicts that drivers will under-report by a constant amount. This is consistent with the idea that drivers have some reservation wage they do not want to fall below. We predict that the monitoring technology will decrease the amount of shading we see in the data. Owners can use the device to estimate actual revenue more accurately, and they are more likely to detect when the driver is underreporting. Drivers should respond by lying less everywhere below the threshold. Figure 13 depicts shading across treatment and control groups, to which we apply a non-parametric smoothing function. We observe constant shading below some threshold value q^* in both groups (which falls somewhere between 500-1000 KES). Moreover, we observe that the treatment group shades less than the control group. To obtain a more precise estimate for the reduction in shading, we regress the shade amount on treatment status for different possible q^* (between 500-1000). The regression only considers data below q^* because this is where the model predicts shading will occur. ¹³ The results in Table 5 confirm that the amount drivers shade falls by approximately 70-100 KES per day depending on the exact location of q^* .

¹²We use gross revenue below average for this outcome instead of net revenue like we did for the shading amount because it only depends on drivers reporting, which means we have more data to work.

¹³The regression includes the standard controls and fixed effects. The regression also excludes data from month 1 because we know that owners were unfamiliar with the device in that first month. The magnitude of the results stay the same when we include month 1 but we lose some precision from the noise this month introduces.

Without knowing q^* exactly, our estimate of 70-100 KES technically includes both a reduction in \tilde{q} and a drop in q^* . We want to confirm that both of these behaviors are indeed happening in reality. We do so by imposing a step function in a regression of shade amount on treatment. In other words we allow under-reporting below q^* and impose zero shading above. We run this regression for every reasonable value of q^* for treatment and control groups. We then plot two outcomes in the second panel of Figure 13. The dots represent the estimated under-reported amount in the treatment (in red) and control (in black) groups across different choices of q^* . We can see that the treatment groups under-report by approximately 50-70 schillings less than the control group regardless of the q^* we impose on the model. Next, we plot the Mean Squared Error (MSE) of our regressions (dotted lines) to isolate the q^* that minimizes the MSE for the treatment and control groups respectively. The vertical lines represent the optimal q^* using this metric. This demonstrates that our best guess of q^* in the treatment group is 150 schillings below our best guess of q^* in the control group. This confirms that both factors explain the overall reduction in under-reporting that we observed in the more flexible regression specification above.

6.3.3 Company Performance and Employee Welfare

We now turn to investigating the impact of the monitoring device on firm performance. Specifically we are interested in determining whether the information we supplied allows companies to generate higher profits and ultimately expand their operations by adding more vehicles to their fleet. Company profits are measured by subtracting costs (repairs and driver salary) from total revenue. We documented substantial reductions in repair costs and, assuming drivers are at their reservation wage, we expect their salary to stay the same (Figure 16 confirms this is true). The model predicts that the impact on revenue, however, is ambiguous. Improved monitoring increases driver effort, and reduces shading. However, it also reduces the amount of risk-taking behavior drivers engage in (which we confirmed in our data). Depending on which of these effects dominates revenue could increase or decrease. Panel 1 of Figure 15 illustrates that revenue does not change substantially throughout the study. Taken together, decreasing costs and stable revenues suggest that firm profits will increase. Panel 2 in Figure 15 demonstrates a similar trend to what we've observed to date: profits increase continuously starting month 3, and peak at month 5. Specifically, treatment owners see their daily profits rise by approximately 12% in month 4 and 5 (440 KES). Taking the average gains over the study period and extrapolating to the full year (assuming the matatu operates 25 days a month), we can expect a 120,000 KES (1200 USD) increase in annual firm profits. It is worth mentioning that this profit measure does not take into account any additional gains from having to spend less time and effort monitoring the driver. The device cost 125 USD (including shipping to Kenya), which means that it would take less than 3 months for the investment to become cost-effective for the owner. This return on investment (ROI) suggests that these devices are likely to be welfare improving for owners in the short and long run. One of the reasons we do not see more matatu owners adopting them, however, is because they currently do not exist in this form on the market. The options are either much more expensive (approximately 600 USD and monthly installments), or have more limited capacity. Without having tested their efficacy, owners are hesitant to make the investment. Our profit gains are in line with some of the more successful business training programs documented in the literature. The cost of these trainings range from 20 to 740 dollars and last a few weeks at most. Our technology has the added benefit of requiring a single up-front payment for continued use. Moreover it requires relatively little coordination and training.

Are treatment firms also more likely to grow their business than control firms? We measure firm growth by the number of vehicles that owners have in their fleet at endline. A simple regression of this outcome on treatment with the standard controls reveals that treatment owners have 0.145 more vehicles in their fleet on average than control owners (Table 7, Column 1). This represents an 11 percentage point increase in fleet size. While treatment owners were also more likely to make changes to their matatu's interior, this result is not statistically significant (Table 7, Column 2). There are a number of reasons why the monitoring device could have encouraged treatment owners to grow their businesses more actively. First, profits increased and shading decreased. Second, our results suggest that owners started trusting their drivers more. Table 6 presents four different measures of owners' perceptions of their drivers at endline. We see owners sending an additional 30 KES to drivers in the trust game the enumerators administered (Column 1). Moreover, treatment owners' assessment of whether their drivers' skills have improved increases by 0.6 points (where they could be assigned a -1 for worse driving, 0 for no change, and 1 for better driving). Finally, treatment owners are more likely to report that their drivers have become more honest (Column 3). We suspect that greater trust in their drivers' abilities/honesty, combined with a reduction in the amount the drivers shade, makes the process of managing the company easier. Together with higher profits, treatment owners may have seen an opportunity to expand that did not exist before.

Finally it is worth investigating whether these gains to the company come at the expense of their employees. While it is difficult to measure welfare, we consider three main outcomes that could impact drivers' well-being: the amount of effort they supply, their salary and their relationship with the owner. We know the amount of effort they supply increases (Figure 11), and the amount they shade decreases (Figure 13). While their salary per hour remains unchanged (Figure 16), they are potentially worse off for working more hours. However, throughout the course of the study we did not receive any complaints from drivers, despite contacting them regularly to conduct our surveys. To investigate this further, we created a small survey that we administered to drivers via SMS 6 months after the original study concluded (at this point we had given control owners 2 months with the information as well so no distinction can be drawn between treatment and control drivers). Sixty percent of drivers responded (distributed evenly across treatment and control) with very positive experiences about the device: 27% said it improved their relationship with the driver (70% said nothing changed), 65% said it made their job easier (26 % said nothing changed), 96% said they preferred driving with the tracker, and 65% said it changed the way they drove. While we do not want to lean too much on this qualitative evidence, it does suggest that the drivers benefitted from the device as well. Some of the open ended questions reveal that drivers felt a greater sense of security with the device in their car, and they felt it increased owners' trust in their work, which reduced the stresses of the job.

6.3.4 Externalities

The device conveyed information to owners about productivity and safety. To the extent that owners contract explicitly over safety we might expect owners to set higher safety standards for their drivers. However, if owners care only about profits, and increased effort comes at the expenses of safety, we might expect instances of unsafe driving to increase. This would lead to socially suboptimal behavior by the drivers. The device collected five pieces of information that correlate with safe driving: maximum speeds, speeding over 80km, acceleration, sharp breaking and sharp turning. We do not see any meaningful increases in maximum or average speeds as the study progresses (Figure 17 reveals a positive trend if anything). Similarly, instances of over-acceleration and sharp breaking do not change significantly (Figure 18 and Figure 19). We see no effect on sharpturns or instances of speeding above 80km (which is difficult in Nairobi to begin with - Figure 20 and Figure 21). Finally we tracked the number of accidents throughout the project. There are 41 accidents in total throughout the 6 month period, of varying degrees of severity. While the number of accidents trends upwards in months 4 and 5, it is difficult to conclude that accidents increase significantly (Figure 22). Overall the evidence points towards safety standards staying the same, despite the emphasis we placed on safety across all tabs in the app. While this confirms that owners can incentivize optimal levels of effort without further compromising passenger safety, we cannot expect owners to internalize the negative externalities produced by unsafe driving.

6.4 Cash Treatment Arm

Finally, we tested the impact of an intervention that incentivizes drivers to take safety into account. Drivers were offered 600 KES at the beginning of the day, and incurred a penalty for each safety violation they incurred. The experiment was designed to mimic an intervention that a regulatory body could feasibly implement. We find that the cash treatment has no discernible effect on average speed, over-acceleration, and sharp turning. However, we detect large decreases in the instances of speeding and sharp braking. The number of sharp braking alerts deceases by 0.13 events per day, a 17% decrease relative to the control group. Likewise, the number of sharp braking events decreases by 0.24 per day, representing a 35% decrease. These results suggest that drivers can be incentivized to take safety into account. However the incentives must come from a third party, as owners are unlikely to induce similar changes in driving behavior.

In Table 9 we examine driving behavior among the group of drivers whose cash incentives were removed after the first month. The goal of this exercise is to examine whether the behavioral changes induced by the cash treatment persist after the incentives are removed. We see that the number of speeding events rebounds almost completely to pre-treatment levels, while the number of sharp braking events remains lower but is statistically insignificant. Overall, it appears that the behavioral effects of the cash treatment arm wear off after the removal of the incentives. This

suggests that inducing better driving habits for a short time period may not be sufficient to see longer run improvement in safety outcomes.

7 Concluding Remarks

In this paper we design a monitoring technology tailored to the minibus industry in Nairobi. The device provides real time information about the productivity and safety of the driver to the owner of the minibus. We find that the monitoring technology eases labor contracting frictions by improving the contract that owners offer their drivers. The drivers respond by supplying more effort, driving in ways that are less damaging to the vehicle, under-reporting revenue by less and making the target more often. This results in higher profits for the firm. Treatment owners also report greater trust in their drivers, and find it less difficult to monitor them, which may explain why their businesses grow faster during the study. Despite the breadth of information we supplied on safety, we do not see drivers improving their performance along this margin unless they are explicitly incentivized to do so with small cash grants. While this suggests that gains to the company do not come at the expense of the quality of service they provide, it also highlights that the technology does not remedy the externalities the industry produces to being with.

These results are important for a number of key stakeholders including small firms operating in the transportation industry, and policy makers working to improve road safety conditions in urban hubs. We know firms struggle to grow in developing countries for a number of reasons, and this paper identifies another important barrier that is relatively understudied empirically: moral hazard in labor contracting. One solution that can potentially ease this friction is improved monitoring. Monitoring is typically difficult in small firms, however, because they cannot hire dedicated staff to oversee employee performance, and it takes time away from regular business operations. In our paper, we demonstrate that introducing cost-effective monitoring technologies can be a worthwhile investment for companies looking to increase their profits and grow their asset base.

We do not find that safety standards improve when information from the device is conveyed to owners. However, when the drivers are incentivized to drive more safely we see instances of speeding and sharp breaking fall. This suggests that simply introducing monitoring technologies, without further regulation, might not achieve the desired effects for governments trying to improve road safety. Local transport authorities in Nairobi and South Africa have already started to discuss ways of introducing remote tracking solutions throughout the transportation industry to help monitor and record the behavior of the drivers on the road. Our research suggests that while this will improve firm operations, more targeted interventions requiring regulatory oversight will be necessary if these devices are to induce safer driving.

This analysis highlights the need for further research estimating the long term impacts of these technologies on firm operations. Our study lasted 6 months, but we hypothesize that we would have seen greater changes in the terms of the contract, and in the *type* of contract being offered had we continued for an additional year. Finally, future research should also investigate how this

information can be used to induce longer-lasting improvements in safe driving.

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Tables

Table 1: Balance across information treatment (owners)

Variable	Control	Treatment	Difference
Install date (days since July 1, 2016)	211.9	212.9	-1.03
			(5.09)
Owner age	36.3	37.3	-0.99
	211.9		(0.99)
Owner gender	0.18	0.18	-0.0056
			(0.048)
Owner highest level of education	2.94	2.97	-0.030
			(0.11)
Owner is employed in salaried job	0.21	0.24	-0.030
			(0.052)
Years the owner is in matatu industry	7.71	7.71	-0.0066
			(0.79)
Years owner has owned matatus	4.65	4.47	0.18
			(0.52)
Number of drivers hired for this matatu	1.26	1.37	-0.12
			(0.13)
Number of other drivers hired in the past	1.77	1.94	-0.17
			(0.22)
Amount given in trust game	117.7	126.2	-8.50
			(12.4)
Owner Raven's score	4.51	4.65	-0.14
			(0.19)
Driver rating: owner's fairness	8.11	8.33	-0.23
			(0.18)

The data are limited to the 250 owners. Asterisks indicate statistical significance at the 1% ***, 5% **, and 10% * levels.

Table 2: Balance across cash treatment (drivers)

Variable	Control	Treatment	Difference
Driver age	34.4	37	-2.64
<u> </u>			$(0.88)^{***}$
Driver highest level of education	2.45	2.48	-0.030
			(0.087)
Driver experience	6.95	8.24	-1.29
			$(0.72)^*$
Weeks unemployed before current job	2.96	2.28	0.68
			(0.77)
Number of vehicles driven for before current	6.05	4.97	1.08
			$(0.57)^*$
Number of past accidents	0.90	0.87	0.035
			(0.13)
Number of months the driver has been employed	15.2	14.3	0.91
			(2.49)
Owner rating: driver's honesty	7.78	7.60	0.18
			(0.18)
Owner rating: how hard driver works	8.29	8.07	0.22
			(0.18)
Owner rating: driver's safety	8.32	8.21	0.11
			(0.18)
Owner rating: driver's performance overall	8.09	8	0.092
			(0.17)
Driver days working for owner	411.7	500.8	-89.2
			(61.2)
Driver Raven's score	4.26	4.28	-0.016
			(0.18)
Revenue at baseline	7744.8	7732.3	12.5
			(207.6)
Baseline target	3113.1	3147.6	-34.5
			(56.6)

The data are limited to the 250 drivers. Asterisks indicate statistical significance at the 1% ***, 5% **, and 10% * levels.

Table 3: Knowledge gathered through the device

	(1)	(2)	(3)
	Know Km	Know Off-route	Know Revenue
Info Treatment	0.268***	0.451***	0.039
	(0.068)	(0.065)	(0.072)
Control Mean of DV	0.47	0.40	0.61
Controls	X	X	X
Matatu N	187	187	187

Each of the variables is a binary indicator for whether the owner knew the number of kilometers, instances of off-route driving and revenue generated by the vehicle. Asterisks indicate statistical significance at the 1% ***, 5% ***, and 10% * levels. Note these questions were added to the end-line survey after the first wave of endlines had already been completed, which is why we only have 187 observations (balanced across treatment and control)

Table 4: Monitoring through the device

	(1)	(2)	(3)	(4)	(5)
	Difficulty Monitor	Monitoring Time	Check (Phone)	Check (Stage)	Check (Third Party)
Info Treatment	-1.845***	-0.721***	0.966	0.184	-0.116
	(0.156)	(0.053)	(0.895)	(0.383)	(0.257)
Control Mean of DV	4.02	-0.01	7.01	1.95	0.95
Controls	X	X	X	X	X
Matatu N	190	190	190	190	190

These variables capture monitoring behaviors by the owner. Difficulty monitoring is an indicator from 1 to 5 for the level of difficulty associated with monitoring (5 = very hard). Change in monitoring captures whether owners are spending less time monitoring (= -1), more time monitoring (= 1), or see no change. Asterisks indicate statistical significance at the 1% ***, 5% **, and 10% * levels. Note these questions were added to the end-line survey after the first wave of endlines had already been completed, which is why we only have 190 observations (balanced across treatment and control)

Table 5: Shading Behavior

	(1) 500	(2) 600	(3) 700	(4) 800	(5) 900	(6) 1000	(7) 1100
Treatment	-63.658^{**} (32.366)	-79.026^{**} (36.311)	-80.957^* (44.088)	-97.075^{**} (46.778)	-93.110^* (47.664)	-81.483^{*} (45.250)	-81.673^{*} (46.821)
Observations	3,378	3,822	4,503	5,339	5,866	6,820	7,101

Table 6: Perceptions of trust

	(1)	(2)	(3)	(4)
	Trust Amount	Better Driving	More Honest	Performance Rating
Info Treatment	33.796**	0.626***	0.708***	0.112
	(15.123)	(0.057)	(0.052)	(0.174)
Control Mean of DV	151.61	0.04	0.04	7.21
Controls	X	X	X	X
Matatu N	244	190	190	246

Asterisks indicate statistical significance at the 1% ***, 5% **, and 10% * levels. The first column represents the amount of KES that was transferred from the owner to the driver in a game of trust. The owner was given an envelope with 900 KES and told that anything they placed back in the envelop would be tripled and sent to the driver. The driver would then choose how much to send back to the owner. The following two columns ask owners whether their drivers have driven better, and become more honest (=+1), or less honest (=-1) in the last 6 months.

Table 7: Business decisions

	(1)	(2)	
	Number Vehicles	New Interior	
Info Treatment	0.145*	0.074	
	(0.078)	(0.057)	
Control Mean of DV	1.22	0.21	
Controls	X	X	
Matatu N	246	240	

Asterisks indicate statistical significance at the 1% ***, 5% **, and 10% * levels.

Table 8: Effect of cash (immediate)

	(1) Average speed	(2) Maximum speed	(3) Speeding	(4) Sharp braking	(5) Overacceleration	(6) Sharp turning
Cash Treatment	-0.099	-0.214	-0.239**	-0.131*	-0.009	0.041
	(0.247)	(0.874)	(0.108)	(0.074)	(0.015)	(0.035)
Mileage in km	0.007	0.022	0.001	0.001	0.000	0.000
	(0.005)	(0.014)	(0.001)	(0.001)	(0.000)	(0.000)
Control Mean of DV	15.89	52.64	0.69	0.77	0.08	0.40
Controls	X	X	X	X	X	X
Matatu FE	X	X	X	X	X	X
Day FE	X	X	X	X	X	X
Route FE	X	X	X	X	X	X
Matatu-Day N	39,072	39,072	39,072	39,072	39,072	39,072

Asterisks indicate statistical significance at the 1% ***, 5% **, and 10% * levels.

Table 9: Effect of no cash (ongoing)

	(1) Average speed	(2) Maximum speed	(3) Speeding	(4) Sharp braking	(5) Overacceleration	(6) Sharp turning
Cash Treatment	-0.120	-0.409	-0.220**	-0.140**	-0.014	-0.000
	(0.220)	(0.756)	(0.096)	(0.058)	(0.013)	(0.031)
One Month Post Treat	-0.039	0.072	-0.058	-0.115	-0.003	-0.017
	(0.260)	(0.971)	(0.135)	(0.089)	(0.012)	(0.031)
Mileage in km	0.008	0.024	0.002	0.001	0.000	0.001
	(0.005)	(0.015)	(0.001)	(0.001)	(0.000)	(0.000)
Control Mean of DV	15.89	52.64	0.69	0.77	0.08	0.40
Controls	X	X	X	X	\mathbf{X}	X
Matatu FE	X	X	X	X	\mathbf{X}	X
Day FE	X	X	X	X	X	X
Route FE	X	X	X	X	X	X
Matatu-Day N	42,405	42,405	42,405	42,405	$42,\!405$	$42,\!405$

Asterisks indicate statistical significance at the 1% ***, 5% **, and 10% * levels.

Figures

Figure 1: Mobile app



(a) Map Viewer



(b) Historical Map Viewer



(c) Safety Feed



(d) Productivity Summary



(e) Report Submit



(f) Report Complete

Figure 2: Study Timeline

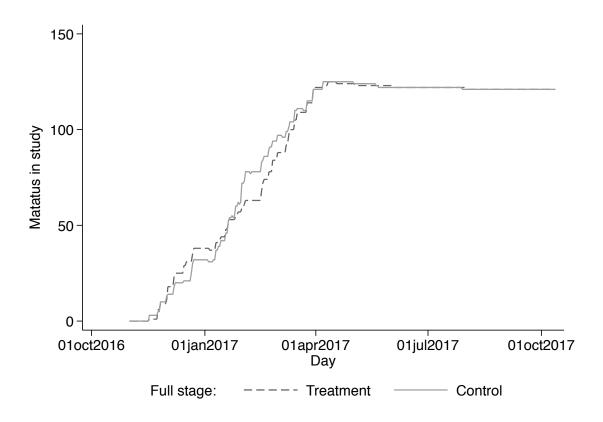
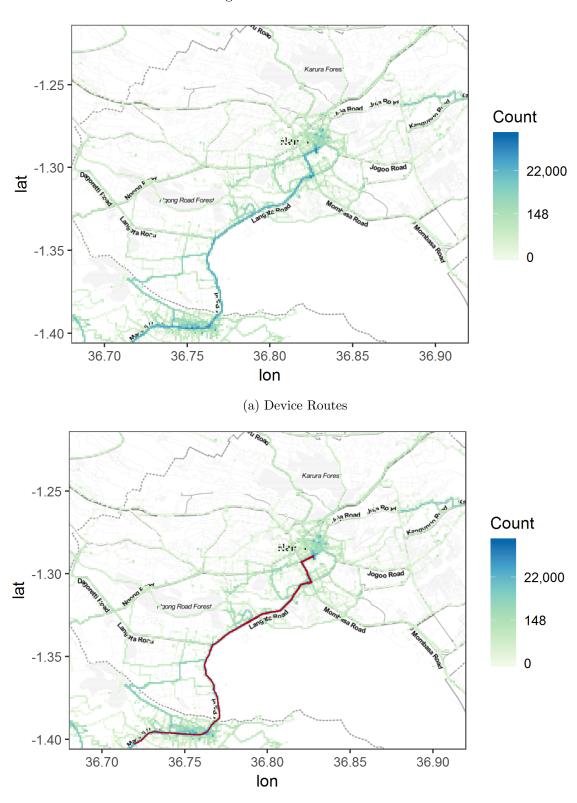


Figure 3: Device Location



(b) Device Routes and Designated Route (Red)

Figure 4: Model Validation (Constant Shading)

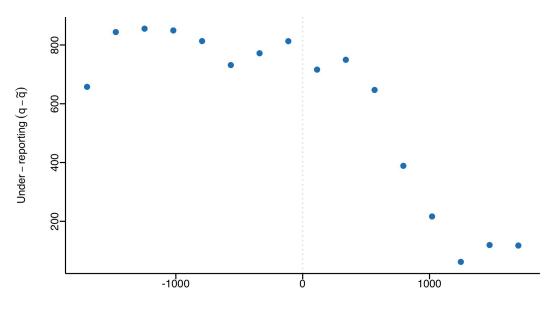
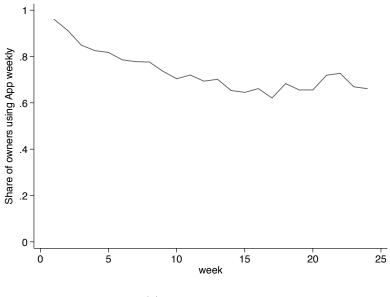
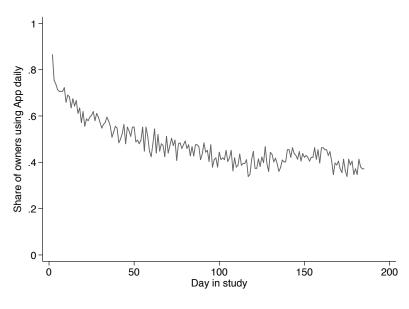


Figure 5: Device Usage: API Calls

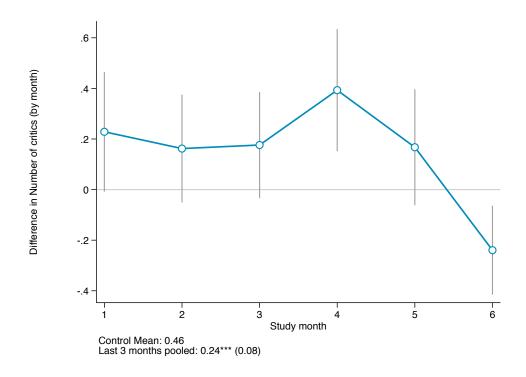


(a) Weekly Usage



(b) Daily Usage

Figure 6: Reprimands and Firing



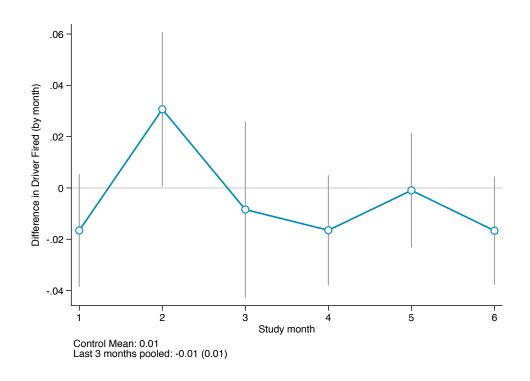


Figure 7: Prediction 1 \rightarrow Target

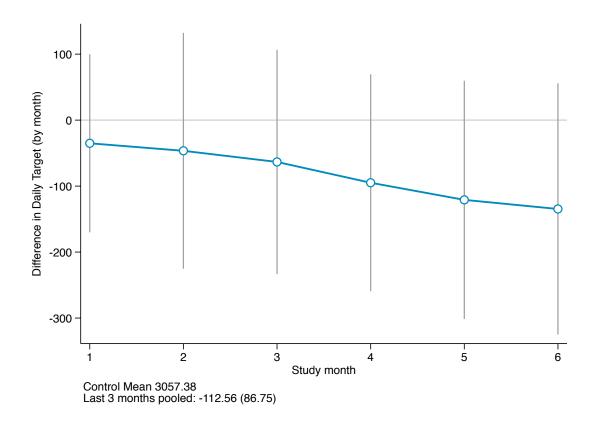
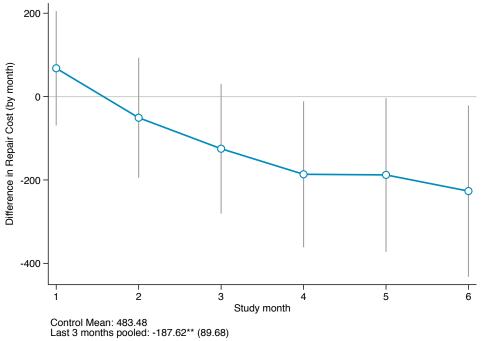


Figure 8: Prediction $2 \to \text{Risk/Damages}$



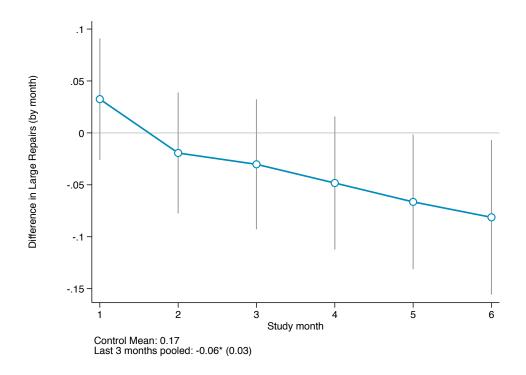
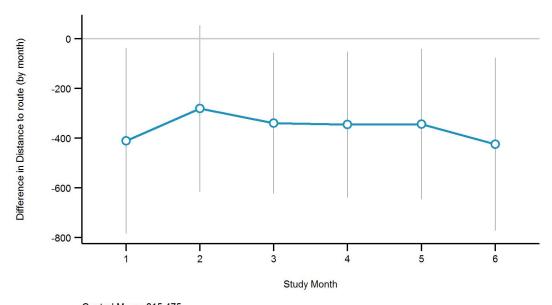
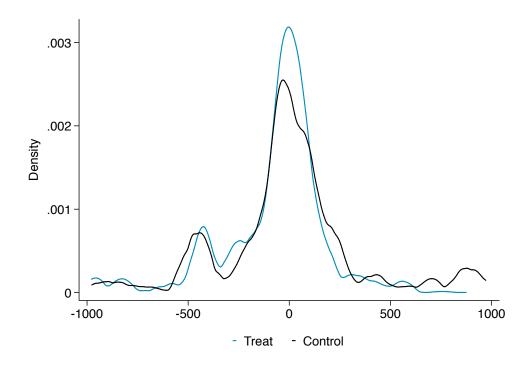


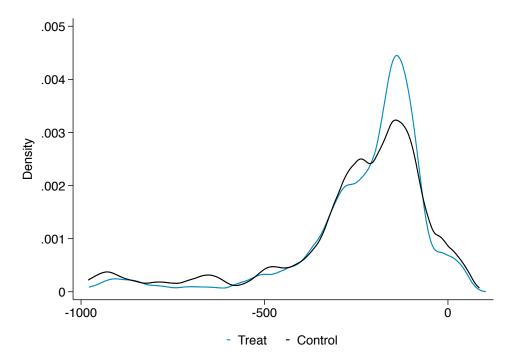
Figure 9: Prediction 2 \rightarrow Risk/Damages



Control Mean: 815.475 Last 3 months pooled: -371.62** (154.251)

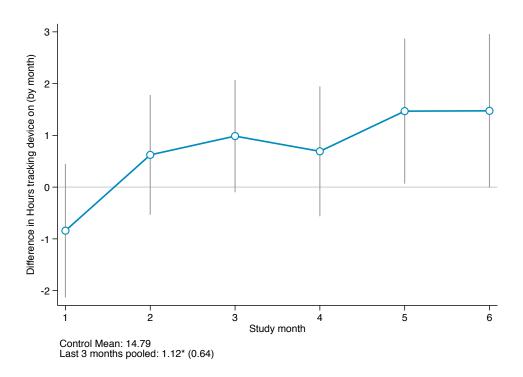
Figure 10: Prediction $2 \to \text{Risk/Damages}$





Notes: It was necessary to downsample the 3-axis acceleration data reported by the tracking devices in order to keep data storage manageable. To do this, several summary statistics were calculated for each 30-second window of acceleration data received. For the vertical component of acceleration, which included both positive and negative readings, the system stored the vector with the maximum magnitude for the window, regardless of direction. To analyze this variable, it was therefore necessary to take the absolute value, resulting in a measure equivalent to the maximum vertical component magnitude within each window. The distribution is centered at -200 rather than 0 (gravity) because of some combination of a non exact calibration and the asymmetry of suspension resulting in asymmetrical acceleration

Figure 11: Prediction $3 \to \text{Effort}$



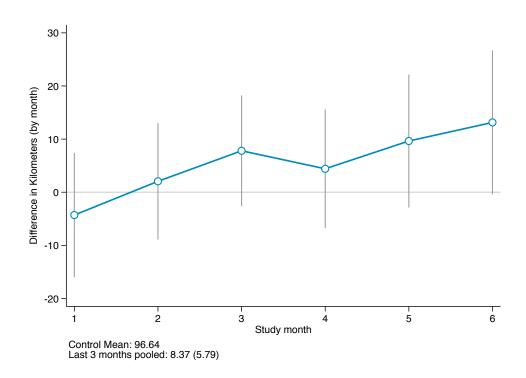
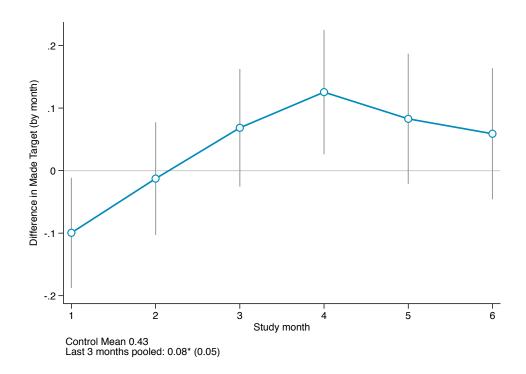


Figure 12: Prediction 5 \rightarrow Achieving Target



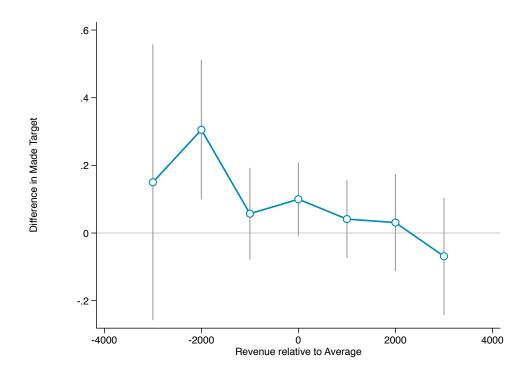
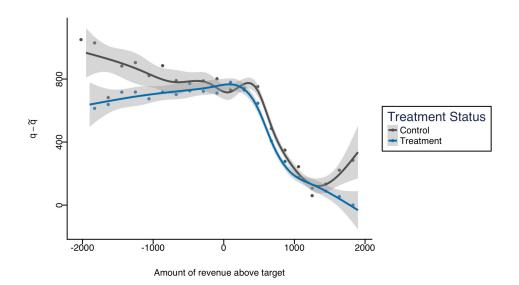


Figure 13: Prediction $4 \to \text{Less Shading}$



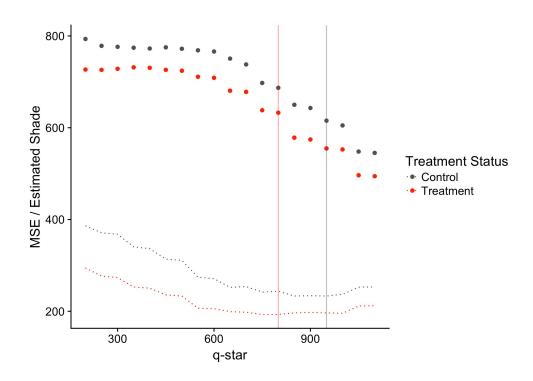


Figure 14: Shade

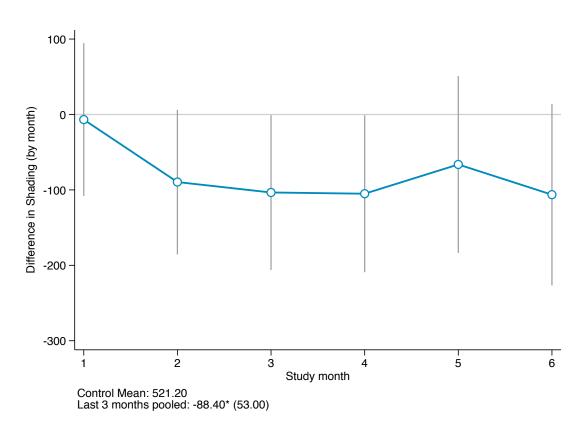
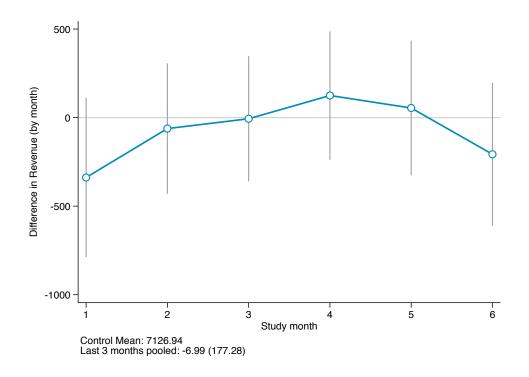


Figure 15: Company Outcomes



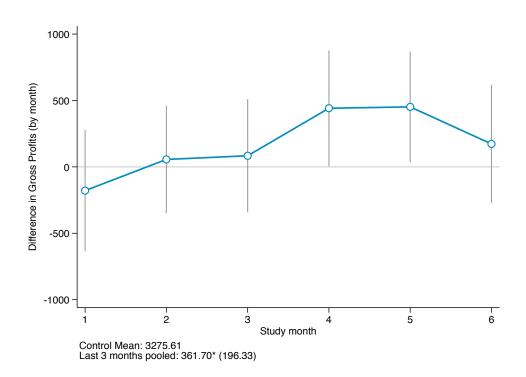
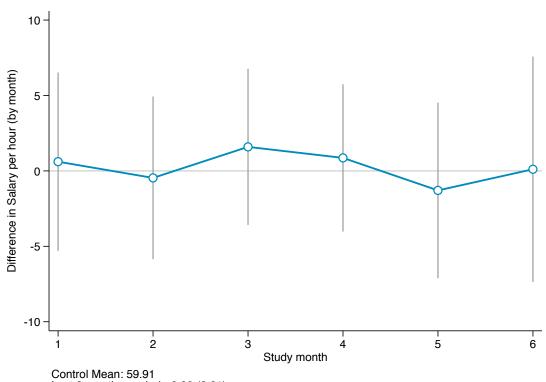
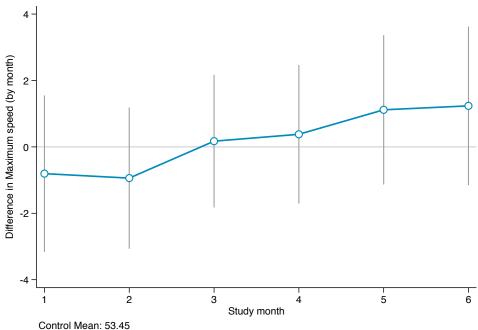


Figure 16: Salary



Control Mean: 59.91 Last 3 months pooled: -0.09 (2.61)

Figure 17: Speeding



Control Mean: 53.45 Last 3 months pooled: 0.84 (1.00)

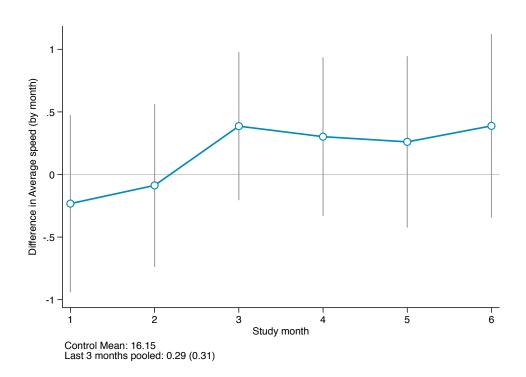


Figure 18: Overacceleration

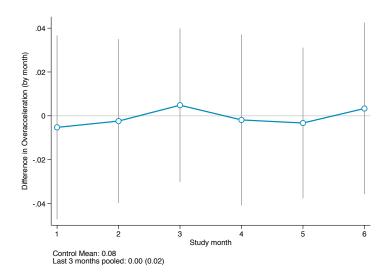


Figure 19: Braking

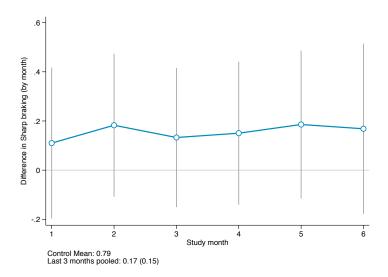


Figure 20: Turning

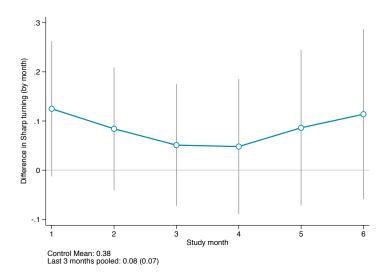


Figure 21: Over-speeding

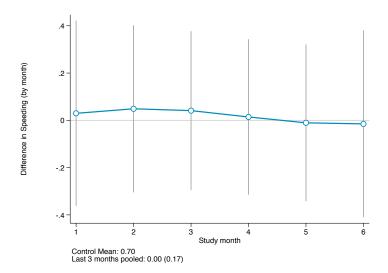
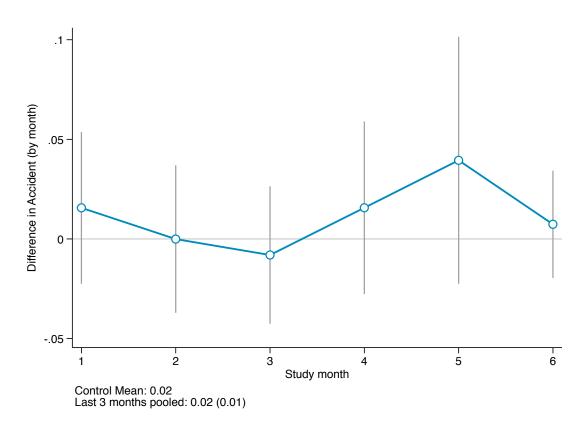


Figure 22: Accidents



Appendix

Step 1: Owner's beliefs

Below we provide the full derivation of the punishment

$$\begin{split} E[punishment] &= E\left[\left(\hat{q} - \frac{1}{\alpha}\right) - \tilde{q} \mid \hat{q} - \frac{1}{\alpha} > \tilde{q}\right] \cdot \Pr\left(\hat{q} - \frac{1}{\alpha} > \tilde{q}\right) \\ &= E\left[\hat{q} - \tilde{q} - \frac{1}{\alpha} \mid \hat{q} > \tilde{q} + \frac{1}{\alpha}\right] \cdot \Pr\left(\hat{q} > \tilde{q} + \frac{1}{\alpha}\right) \\ &= \int_{\tilde{q} + \frac{1}{\alpha}}^{q + \frac{1}{\alpha}} \left(\hat{q} - \tilde{q} - \frac{1}{\alpha}\right) \cdot f(\hat{q}) d\hat{q} \\ &= \frac{\alpha}{2} \int_{\tilde{q} + \frac{1}{\alpha}}^{q + \frac{1}{\alpha}} \left(\hat{q} - \tilde{q} - \frac{1}{\alpha}\right) d\hat{q} \\ &= \frac{\alpha}{2} \left[\frac{\hat{q}^2}{2} - \left(\tilde{q} + \frac{1}{\alpha}\right)\hat{q}\Big|_{\tilde{q} + \frac{1}{\alpha}}^{q + \frac{1}{\alpha}}\right] \\ &= \frac{\alpha}{2} \left[\frac{1}{2} \left(q + \frac{1}{\alpha}\right)^2 - \left(\tilde{q} + \frac{1}{\alpha}\right) \left(q + \frac{1}{\alpha}\right) - \frac{1}{2} \left(\tilde{q} + \frac{1}{\alpha}\right)^2 + \left(\tilde{q} + \frac{1}{\alpha}\right)^2\right] \\ &= \frac{\alpha}{2} \left[\frac{1}{2} \left(q + \frac{1}{\alpha}\right)^2 - \left(\tilde{q} + \frac{1}{\alpha}\right) \left(q + \frac{1}{\alpha}\right) + \left(\tilde{q} + \frac{1}{\alpha}\right)^2\right] \\ &= \frac{\alpha}{4} \left[\left(q + \frac{1}{\alpha}\right)^2 - 2 \left(\tilde{q} + \frac{1}{\alpha}\right) \left(q + \frac{1}{\alpha}\right) + \left(\tilde{q} + \frac{1}{\alpha}\right)^2\right] \\ &= \frac{\alpha}{4} \left[\left(q + \frac{1}{\alpha}\right)^2 - \left(\tilde{q} + \frac{1}{\alpha}\right)^2\right] \\ &= \frac{\alpha}{4} \left(q - \tilde{q}\right)^2 \end{split}$$

Step 2: Solve the agent's optimal shading amount

Below we provide the full derivation of the optimal shading amount

$$\begin{split} \frac{\partial U^D}{\partial \tilde{q}} &= -1 + \frac{\alpha}{2}(q - \tilde{q}) = 0 \\ &\frac{\alpha}{2}(q - \tilde{q}) = \frac{1}{2} \\ &q - \tilde{q} = \frac{2}{\alpha} \\ &- \tilde{q} = -q + \frac{2}{\alpha} \\ &\tilde{q} - q = \frac{2}{\alpha} \end{split}$$

Step 3: Switch point

Below we provide the full derivation of the switch point

$$q - T - \beta r = (q - \tilde{q}) - \frac{\alpha}{4}(q - \tilde{q})^2 - \beta r$$

$$q - T = \left(q - q + \frac{2}{\alpha}\right) - \frac{\alpha}{4}\left(q - q + \frac{2}{\alpha}\right)^2$$

$$q - T = \frac{2}{\alpha} - \frac{\alpha}{4}\left(\frac{4}{\alpha^2}\right)$$

$$q - T = \frac{1}{\alpha}$$

$$q^* = T + \frac{1}{\alpha}$$

Step 4: Driver's optimal choice of effort and risk

The driver chooses effort to maximize his utility

$$\max_{e,r} \quad E[(q - T - \beta r) \mid q \ge q^*] \cdot Pr(q \ge q^*) + E[(q - \tilde{q}) - \frac{\alpha}{4}(q - \tilde{q})^2 - \beta r \mid q < q^*] \cdot Pr(q < q^*) - h(e, r)$$

Simplifying

$$\max_{e,r} \quad E\left[(q - T - \beta r) \mid q \ge q^* \right] (1 - F(q^*)) + E\left[\frac{1}{\alpha} - \beta r \mid q < q^* \right] \cdot F(q^*) - h(e, r)$$

Expressing in terms of exogenous variable

$$\max_{e,r} \quad E\left[\left(e+r\varepsilon-T-\beta r\right)\mid \varepsilon \geq \frac{q^*-e}{r}\right] \cdot \left(1-F_\varepsilon\left(\frac{q^*-e}{r}\right)\right) + E\left[\frac{1}{\alpha}-\beta r\mid \varepsilon < \frac{q^*-e}{r}\right] \cdot F_\varepsilon\left(\frac{q^*-e}{r}\right) - h(e,r)$$

Using integral notation:

$$L = \int_{\frac{q^* - e}{r}}^{\infty} (e + r\varepsilon - T - \beta r) f_{\varepsilon}(\varepsilon) d\varepsilon + \int_{0}^{\frac{q^* - e}{r}} \left(\frac{1}{\alpha} - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon - h(e, r)$$

Taking the derivative with respect to e

$$\frac{\partial L}{\partial e} = \int_{\frac{q^* - e}{r}}^{\infty} 1 \cdot f_{\varepsilon}(\varepsilon) d\varepsilon + \frac{1}{r} \frac{(q^* - T - \beta r) f_{\varepsilon}}{(q^* - T - \beta r) f_{\varepsilon}} \left(\frac{q^* - e}{r}\right) + 0 - \frac{1}{r} \left(\frac{1}{\alpha} - \beta r\right) f_{\varepsilon} \left(\frac{q^* - e}{r}\right) - h'_{e}$$

$$= \int_{\frac{q^* - e}{r}}^{\infty} f_{\varepsilon}(\varepsilon) d\varepsilon - h'_{e}$$

$$\to \underbrace{1 - F_{\varepsilon}}_{FOC} \left(\frac{q^* - e}{r}\right) - h'_{e} = 0$$

Taking the derivative with respect to r

$$\begin{split} \frac{\partial L}{\partial r} &= \int_{\frac{q^*-e}{r}}^{\infty} \left(\varepsilon - \beta\right) \cdot f_{\varepsilon}(\varepsilon) d\varepsilon + \underbrace{\left(\frac{q^*-e}{r^2}\right) \left(q^* - T - \beta r\right) f_{\varepsilon}\left(\frac{q^*-e}{r}\right)}_{r} + \\ & \int_{0}^{\frac{q^*-e}{r}} \left(-\beta\right) \cdot f_{\varepsilon}(\varepsilon) d\varepsilon - \underbrace{\left(\frac{q^*-e}{r^2}\right) \left(\frac{1}{\alpha} - \beta r\right) f_{\varepsilon}\left(\frac{q^*-e}{r}\right)}_{r} - h'_{r} \\ &= \int_{\frac{q^*-e}{r}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon - h'_{r} - \beta \left(\int_{\frac{q^*-e}{r}}^{\infty} f_{\varepsilon}(\varepsilon) d\varepsilon + \int_{0}^{\frac{q^*-e}{r}} f_{\varepsilon}(\varepsilon) d\varepsilon\right) \\ & \to \underbrace{\int_{\frac{q^*-e}{r}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon - h'_{r} - \beta}_{F,O,C} = 0 \end{split}$$

Next we investigate how a change in T affects effort and risk:

$$\begin{bmatrix} \frac{\partial e}{\partial T} \\ \frac{\partial r}{\partial T} \end{bmatrix} = -\begin{bmatrix} \frac{\partial^2 L}{\partial e^2} & \frac{\partial L}{\partial r \partial e} \\ \frac{\partial L}{\partial e \partial r} & \frac{\partial^2 L}{\partial r^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial T \partial e} \\ \frac{\partial L}{\partial T \partial r} \end{bmatrix}$$

$$= -\frac{1}{\underbrace{\text{Determinant}}} \begin{bmatrix} \frac{\partial^2 L}{\partial r^2} & -\frac{\partial L}{\partial r \partial e} \\ -\frac{\partial L}{\partial e \partial r} & \frac{\partial^2 L}{\partial e^2} \end{bmatrix} \begin{bmatrix} \frac{\partial L}{\partial T \partial e} \\ \frac{\partial L}{\partial T \partial r} \end{bmatrix}$$

$$SOC \text{ for Hessian } > 0$$

Taking each term in turn:

$$\frac{\partial^{2}L}{\partial r^{2}} = 0 - \frac{\partial}{\partial r} \left(\frac{q^{*} - e}{r}\right) \left(\frac{q^{*} - e}{r}\right) f_{\varepsilon} \left(\frac{q^{*} - e}{r}\right) - h_{rr}'' - 2\beta$$

$$= \left(\frac{q^{*} - e}{r^{2}}\right) \left(\frac{q^{*} - e}{r}\right) f_{\varepsilon} \left(\frac{q^{*} - e}{r}\right) - h_{rr}'' - 2\beta$$

$$= \frac{1}{r} \left(\frac{q^{*} - e}{r}\right)^{2} f_{\varepsilon}(\cdot) - h_{rr}''$$

$$S.O.C < 0$$

$$\frac{\partial^{2}L}{\partial e^{2}} = f_{\varepsilon}(\cdot) \left(\frac{1}{r}\right) - h_{ee}''$$

$$= \frac{1}{r} f_{\varepsilon}(\cdot) - h_{ee}''$$

$$S.O.C < 0$$

$$\frac{\partial L}{\partial e \partial r} = 0 - \left[-\frac{1}{r} \left(\frac{q^{*} - e}{r}\right) f_{\varepsilon} \left(\frac{q^{*} - e}{r}\right)\right] - h_{er}''$$

$$= \underbrace{\left(\frac{q^{*} - e}{r^{2}}\right) f_{\varepsilon}(\cdot) - h_{er}''}_{< 0}}_{< 0}$$

$$\frac{\partial L}{\partial r \partial e} = \underbrace{-f_{\varepsilon}(\cdot)} \left(\frac{q^{*} - e}{r^{2}}\right) - h_{er}''$$

$$= \underbrace{0}$$

$$\frac{\partial L}{\partial T \partial r} = 0 - \frac{1}{r} f_{\varepsilon} \left(\frac{q^{*} - e}{r}\right)$$

$$= \underbrace{-\frac{1}{r} f_{\varepsilon}(\cdot)}_{< 0}$$

$$\frac{\partial L}{\partial T \partial r} = 0 - \frac{1}{r} \left(\frac{q^{*} - e}{r}\right) f_{\varepsilon} \left(\frac{q^{*} - e}{r}\right)$$

$$= \underbrace{-\left(\frac{q^{*} - e}{r^{2}}\right) f_{\varepsilon}(\cdot)}_{> 0}$$

We can sign most of these terms because of 1) second order conditions and 2) the shape of the distribution of revenue (q), which is skewed to the left, and the fact that drivers make the target 44% of the time. This means $(q^* - e < 0)$. Note, we would expect the cross partial to be negative because as the driver increases risk (fatter tails), they are less likely to make the target, which

means the returns to making the target decrease and effort will be reduced. Putting it altogether:

$$\frac{\partial e}{\partial T} = -\underbrace{\frac{1}{\text{Determinant}}}_{+} \left[\underbrace{\frac{\partial^{2} L}{\partial r^{2}} \cdot \underbrace{\frac{\partial L}{\partial T \partial e}}_{-} - \underbrace{\frac{\partial L}{\partial r \partial e}}_{-} \cdot \underbrace{\frac{\partial L}{\partial T \partial r}}_{-} \right]$$

$$< 0$$

$$\frac{\partial r}{\partial T} = -\underbrace{\frac{1}{\text{Determinant}}}_{+} \left[-\underbrace{\frac{\partial L}{\partial e \partial r} \cdot \underbrace{\frac{\partial L}{\partial T \partial e}}_{-} + \underbrace{\frac{\partial^{2} L}{\partial e^{2}} \cdot \underbrace{\frac{\partial L}{\partial T \partial r}}_{-}}_{-} + \underbrace{\frac{\partial L}{\partial e^{2}} \cdot \underbrace{\frac{\partial L}{\partial T \partial r}}_{-} \right]$$

$$> 0$$

Next we investigate how a change in α affects effort and risk:

Computing the additional terms

$$\frac{\partial L}{\partial \alpha \partial e} = f_{\varepsilon} \left(\frac{q^* - e}{r} \right) \left(\frac{1}{r \cdot \alpha^2} \right)$$

$$= \frac{1}{\alpha^2 \cdot r} f_{\varepsilon}(\cdot)$$

$$\frac{\partial L}{\partial \alpha \partial r} = 0 + \frac{1}{r \cdot 4\alpha^2} \left(\frac{q^* - e}{r} \right) f_{\varepsilon} \left(\frac{q^* - e}{r} \right)$$

$$= \frac{1}{\alpha^2 \cdot r} \left(\frac{q^* - e}{r} \right) f_{\varepsilon}(\cdot)$$

Putting it altogether:

$$\frac{\partial e}{\partial \alpha} = -\underbrace{\frac{1}{\text{Determinant}}}_{+} \left[\underbrace{\frac{\partial^{2} L}{\partial r^{2}} \cdot \underbrace{\frac{\partial L}{\partial \alpha \partial e}}_{-} - \underbrace{\frac{\partial L}{\partial r \partial e}}_{-} \cdot \underbrace{\frac{\partial L}{\partial \alpha \partial r}}_{-} \underbrace{\frac{\partial L}{\partial \alpha \partial r}}_{-} \right]$$

$$> 0$$

$$\frac{\partial r}{\partial \alpha} = -\underbrace{\frac{1}{\text{Determinant}}}_{+} \left[-\underbrace{\frac{\partial L}{\partial e \partial r} \cdot \underbrace{\frac{\partial L}{\partial \alpha \partial e}}_{-} + \underbrace{\frac{\partial^{2} L}{\partial e^{2}} \cdot \underbrace{\frac{\partial L}{\partial \alpha \partial r}}_{-}}_{-} \underbrace{\frac{\partial L}{\partial \alpha \partial r}}_{-} \right]$$

$$< 0$$

Finally we investigate how a change in β affects effort and risk:

$$\begin{bmatrix} \frac{\partial e}{\partial \beta} \\ \frac{\partial r}{\partial \beta} \end{bmatrix} = -\begin{bmatrix} \frac{\partial^2 L}{\partial e^2} & \frac{\partial L}{\partial r \partial e} \\ \frac{\partial L}{\partial e \partial r} & \frac{\partial^2 L}{\partial r^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial \beta \partial e} \\ \frac{\partial L}{\partial \beta \partial r} \end{bmatrix}$$

$$= -\frac{1}{\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{Determinant}}}}_{S.O.C \text{ for Hessian}} > 0}}} \begin{bmatrix} \frac{\partial^2 L}{\partial r^2} & -\frac{\partial L}{\partial r \partial e} \\ -\frac{\partial L}{\partial e \partial r} & \frac{\partial^2 L}{\partial e^2} \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Putting it altogether:

$$\frac{\partial e}{\partial \beta} = -\underbrace{\frac{1}{\text{Determinant}}}_{+} \left[\underbrace{\frac{\partial L}{\partial r \partial e}}_{+} \cdot \underbrace{\frac{+}{1}}_{1} \right]$$

$$> 0$$

$$\frac{\partial r}{\partial \beta} = -\underbrace{\frac{1}{\text{Determinant}}}_{+} \left[\underbrace{\frac{-}{\partial^{2} L}}_{+} \cdot \underbrace{(-1)}_{-1} \right]$$

$$< 0$$

Step 5: Owner's optimal reporting choice

Constrained case

The owner chooses T to maximize his utility:

$$\begin{aligned} & \max_{T} & T \cdot Pr(q \geq q^*) + E[\tilde{q} \mid q < q^*] \cdot Pr(q < q^*) - \gamma(r) & \text{s.t} \\ & E\left[q - T - \beta r \mid q \geq q^*\right] \cdot Pr(q \geq q^*) + E\left[q - \tilde{q} - \frac{\alpha}{4}(q - \tilde{q})^2 - \beta r \mid q < q^*\right] \cdot Pr(q < q^*) - h(e^*, r^*) > R \end{aligned}$$

Expressing in terms of exogenous variables:

$$\begin{aligned} & \max_{T} & T \cdot Pr\left(\varepsilon \geq \frac{q^* - e^*}{r}\right) + E\left[e + r\varepsilon - \frac{1}{2\alpha} \mid \varepsilon < \frac{q^* - e^*}{r}\right] \cdot Pr\left(\varepsilon < \frac{q^* - e^*}{r}\right) - \gamma(r) & \text{s.t.} \\ & E\left[\left(e + r\varepsilon - T - \beta r\right) \mid \varepsilon \geq \frac{q^* - e}{r}\right] \cdot Pr(\varepsilon \geq \left(\frac{q^* - e}{r}\right) + E\left[\frac{1}{\alpha} - \beta r \mid \varepsilon < \frac{q^* - e}{r}\right] \cdot Pr\left(\varepsilon < \frac{q^* - e}{r}\right) - h(e, r) \geq 0 \end{aligned}$$

Translating into integral notation:

$$L = \underbrace{T \int_{\frac{q^* - e^*}{r^*}}^{\infty} f_{\varepsilon}(\varepsilon) d\varepsilon}_{A} + \underbrace{\int_{0}^{\frac{q^* - e^*}{r^*}} \left(e + r\varepsilon - \frac{1}{2\alpha}\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{B} - \gamma(r) + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{B} + \underbrace{\int_{0}^{\frac{q^* - e^*}{r^*}} \left(\frac{1}{\alpha} - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} - \lambda(e, r) + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon - T - \beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{D} + \underbrace{\int_{0}^{\infty} \left(e + r\varepsilon$$

Taking the derivative with respect to T

$$=\underbrace{\int_{\frac{q^*-e^*}{r^*}}^{\infty}f_{\varepsilon}(\varepsilon)d\varepsilon+T\left[0-\frac{\partial}{\partial T}\left(\frac{q^*-e^*}{r^*}\right)f_{\varepsilon}(\cdot)\right]}_{A'}+\underbrace{\int_{0}^{\frac{q^*-e^*}{r^*}}\left(\frac{\partial e}{\partial T}+\frac{\partial r}{\partial T}\varepsilon\right)f_{\varepsilon}(\varepsilon)d\varepsilon+\frac{\partial}{\partial T}\left(\frac{q^*-e^*}{r^*}\right)\left(T-\frac{1}{\alpha}\right)f_{\varepsilon}(\cdot)}_{B'}-\gamma'(r)\frac{\partial r}{\partial T}+\lambda\left[\underbrace{\int_{\frac{q^*-e^*}{r^*}}^{\infty}\left(\frac{\partial e}{\partial T}+\frac{\partial r}{\partial T}\varepsilon-1-\left(\beta\cdot\frac{\partial r}{\partial T}\right)\right)f_{\varepsilon}(\varepsilon)d\varepsilon-\frac{\partial}{\partial T}\left(\frac{q^*-e^*}{r^*}\right)\frac{1}{\alpha}f_{\varepsilon}(\cdot)}_{C'}\right]}_{C'}+\underbrace{\underbrace{\int_{0}^{\frac{q^*-e^*}{r^*}}\left(\frac{\partial e}{\partial T}+\frac{\partial r}{\partial T}\varepsilon-1-\left(\beta\cdot\frac{\partial r}{\partial T}\right)\right)f_{\varepsilon}(\varepsilon)d\varepsilon-\frac{\partial}{\partial T}\left(\frac{q^*-e^*}{r^*}\right)\frac{1}{\alpha}f_{\varepsilon}(\cdot)}_{C'}}_{C'}$$

$$=\underbrace{1-F_{\varepsilon}(\cdot)-T\frac{\partial}{\partial T}\underbrace{\left(\frac{q^{*}-e^{*}}{r^{*}}\right)f_{\varepsilon}(\cdot)}_{A'}+\underbrace{\frac{\partial e}{\partial T}F_{\varepsilon}(\cdot)+\frac{\partial r}{\partial T}\int_{0}^{\frac{q^{*}-e^{*}}{r^{*}}}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon+\underbrace{\frac{\partial}{\partial T}\underbrace{\left(\frac{q^{*}-e^{*}}{r^{*}}\right)\left(T-\frac{1}{\alpha}\right)f_{\varepsilon}(\cdot)}_{B'}-\gamma'(r)\frac{\partial r}{\partial T}}_{B'}$$

$$+\lambda\left[\underbrace{-(1-F_{\varepsilon}(\cdot))+\frac{\partial e}{\partial T}(1-F_{\varepsilon}(\cdot))+\frac{\partial r}{\partial T}\int_{\frac{q^{*}-e^{*}}{r^{*}}}^{\infty}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon-\underbrace{\frac{\partial}{\partial T}\underbrace{\left(\frac{q^{*}-e^{*}}{r^{*}}\right)\frac{1}{\alpha}f_{\varepsilon}(\cdot)}_{C'}}_{C'}\right]$$

$$+\underbrace{\frac{\partial}{\partial T}\underbrace{\left(\frac{q^{*}-e^{*}}{r^{*}}\right)\frac{1}{\alpha}f_{\varepsilon}(\cdot)-h'\left(\frac{\partial e}{\partial T}+\frac{\partial r}{\partial T}\right)-\beta\frac{\partial r}{\partial T}}_{D'}}_{D'}$$

$$=\underbrace{1-F_{\varepsilon}(\cdot)}_{A'} + \underbrace{\frac{\partial e}{\partial T}F_{\varepsilon}(\cdot) + \frac{\partial r}{\partial T} \int_{0}^{\frac{q^*-e^*}{r^*}} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon - \frac{\partial}{\partial T} \left(\frac{q^*-e^*}{r^*}\right) \left(\frac{1}{\alpha}\right) f_{\varepsilon}(\cdot)}_{B'} - \gamma'(r) \frac{\partial r}{\partial T}$$

$$+ \lambda \left[\underbrace{-(1-F_{\varepsilon}(\cdot)) + \frac{\partial e}{\partial T} (1-F_{\varepsilon}(\cdot)) + \frac{\partial r}{\partial T} \int_{\frac{q^*-e^*}{r^*}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon + \frac{\partial r}{\partial T} (-\beta) - \underbrace{h'\left(\frac{\partial e}{\partial T} + \frac{\partial r}{\partial T}\right)}_{D'}\right] = 0$$

Taking the derivative with respect to λ

$$\int_{\frac{q^*-e^*}{r^*}}^{\infty} \left(e+r\varepsilon-T-\beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon + \int_{0}^{\frac{q^*-e^*}{r^*}} \left(\frac{1}{\alpha}-\beta r\right) f_{\varepsilon}(\varepsilon) d\varepsilon - h(e,r) = 0$$

Next we apply the IFT to understand how T changes with α .

$$\begin{bmatrix} \frac{\partial T}{\partial \alpha} \\ \frac{\partial \lambda}{\partial \bar{\alpha}} \end{bmatrix} = -\begin{bmatrix} \frac{\partial^2 L}{\partial T^2} & \frac{\partial L}{\partial \lambda \partial T} \\ \frac{\partial L}{\partial T \partial \lambda} & \frac{\partial^2 L}{\partial \lambda^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial \alpha \partial T} \\ \frac{\partial L}{\partial \alpha \partial \lambda} \end{bmatrix}$$

$$= -\frac{1}{\text{Determinant}} \begin{bmatrix} \frac{\partial^2 L}{\partial \lambda^2} & -\frac{\partial L}{\partial \lambda \partial T} \\ -\frac{\partial L}{\partial T \partial \lambda} & \frac{\partial^2 L}{\partial T^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial \alpha \partial T} \\ \frac{\partial L}{\partial \alpha \partial \lambda} \end{bmatrix}$$

$$= -\frac{1}{\text{Determinant}} \begin{bmatrix} 0 & -\frac{\partial L}{\partial \lambda \partial T} \\ -\frac{\partial L}{\partial T \partial \lambda} & \frac{\partial^2 L}{\partial T^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial \alpha \partial T} \\ \frac{\partial L}{\partial \alpha \partial \lambda} \end{bmatrix}$$

Taking each term in turn:

$$\begin{split} \frac{\partial L}{\partial \lambda \partial T} &= -(1 - F_{\varepsilon}(\cdot)) + \frac{\partial e}{\partial T}(1 - F_{\varepsilon}(\cdot)) + \frac{\partial r}{\partial T} \int_{\frac{q^* - e^*}{r^*}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon + \frac{\partial r}{\partial T}(-\beta) - h' \left(\frac{\partial e}{\partial T} + \frac{\partial r}{\partial T}\right) \\ &= -(1 - F_{\varepsilon}(\cdot)) + \underbrace{\frac{\partial e}{\partial T} \left(1 - F_{\varepsilon}(\cdot) - h'_{e}\right)}_{F.O.C = 0} + \underbrace{\frac{\partial r}{\partial T} \left(\int_{\frac{q^* - e^*}{r^*}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon - h'_{r} - \beta\right)}_{F.O.C = 0} \\ &= \underbrace{-(1 - F_{\varepsilon}(\cdot))}_{c.0} \\ \frac{\partial L}{\partial \alpha \partial \lambda} &= \int_{\frac{q^* - e^*}{r^*}}^{\infty} \left(\frac{\partial e}{\partial \alpha} + \frac{\partial r}{\partial \alpha} \varepsilon - \beta \frac{\partial r}{\partial \alpha}\right) f_{\varepsilon}(\varepsilon) d\varepsilon - \underbrace{\frac{\partial e}{\partial \alpha} \left(\frac{q^* - e^*}{r^*}\right) \left(\frac{1}{\alpha} - \beta r\right) f_{\varepsilon}(\cdot)}_{c.0} - h' \left(\frac{\partial e}{\partial \alpha} + \frac{\partial r}{\partial \alpha}\right) \\ &- \int_{0}^{\frac{q^* - e^*}{r^*}} \left(\frac{1}{\alpha^2} + \beta \frac{\partial r}{\partial \alpha}\right) f_{\varepsilon}(\varepsilon) d\varepsilon + \underbrace{\frac{\partial e}{\partial \alpha} \left(\frac{q^* - e^*}{r^*}\right) \left(\frac{1}{\alpha} - \beta r\right) f_{\varepsilon}(\cdot)}_{c.0} - h' \left(\frac{\partial e}{\partial \alpha} + \frac{\partial r}{\partial \alpha}\right) \\ &= \int_{\frac{q^* - e^*}{r^*}}^{\infty} \left(\frac{\partial e}{\partial \alpha} + \frac{\partial r}{\partial \alpha} \varepsilon - \beta \frac{\partial r}{\partial \alpha}\right) f_{\varepsilon}(\varepsilon) d\varepsilon - \int_{0}^{\frac{q^* - e^*}{r^*}} \left(\frac{1}{\alpha^2} + \beta \frac{\partial r}{\partial \alpha}\right) f_{\varepsilon}(\varepsilon) d\varepsilon - h' \left(\frac{\partial e}{\partial \alpha} + \frac{\partial r}{\partial \alpha}\right) \\ &= \frac{\partial e}{\partial \alpha} (1 - F_{\varepsilon}(\cdot)) + \frac{\partial r}{\partial \alpha} \int_{\frac{q^* - e^*}{r^*}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon - \int_{0}^{\frac{q^* - e^*}{r^*}} \left(\frac{1}{\alpha^2}\right) f_{\varepsilon}(\varepsilon) d\varepsilon - h' \left(\frac{\partial e}{\partial \alpha} + \frac{\partial r}{\partial \alpha}\right) - \beta \frac{\partial r}{\partial \alpha} \\ &= \underbrace{\frac{\partial e}{\partial \alpha} (1 - F_{\varepsilon}(\cdot) - h'_{e})}_{F.O.C = 0} + \underbrace{\frac{\partial r}{\partial \alpha} \left(\int_{\frac{q^* - e^*}{r^*}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon - h'_{r} - \beta\right)}_{F.O.C = 0} - \int_{0}^{\frac{q^* - e^*}{r^*}} \left(\frac{1}{\alpha^2}\right) f_{\varepsilon}(\varepsilon) d\varepsilon \\ &= \underbrace{\frac{\partial e}{\partial \alpha} (1 - F_{\varepsilon}(\cdot) - h'_{e})}_{F.O.C = 0} + \underbrace{\frac{\partial r}{\partial \alpha} \left(\int_{\frac{q^* - e^*}{r^*}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon - h'_{r} - \beta\right)}_{F.O.C = 0} - \int_{0}^{\frac{q^* - e^*}{r^*}} \left(\frac{1}{\alpha^2}\right) f_{\varepsilon}(\varepsilon) d\varepsilon \\ &= \underbrace{\frac{\partial e}{\partial \alpha} (1 - F_{\varepsilon}(\cdot) - h'_{e})}_{F.O.C = 0} + \underbrace{\frac{\partial r}{\partial \alpha} \left(\int_{\frac{q^* - e^*}{r^*}}^{\infty} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon - h'_{r} - \beta\right)}_{F.O.C = 0} - \underbrace{\frac{\partial r}{\partial \alpha} \left(\frac{\partial r}{\partial \alpha} + \frac{\partial r}{\partial \alpha}\right)}_{F.O.C = 0} + \underbrace{\frac{\partial r}{\partial \alpha} \left(\frac{\partial r}{\partial \alpha} + \frac{\partial r}{\partial \alpha}\right)}_{F.O.C = 0} + \underbrace{\frac{\partial r}{\partial \alpha} \left(\frac{\partial r}{\partial \alpha} + \frac{\partial r}{\partial \alpha}\right)}_{F.O.C = 0} + \underbrace{\frac{\partial r}{\partial \alpha} \left(\frac{\partial r}{\partial \alpha} + \frac{\partial r}{\partial \alpha}\right)}_{F.O.C = 0} + \underbrace{\frac{\partial r}{\partial \alpha} \left(\frac{\partial r}{\partial \alpha} + \frac{\partial r}{\partial \alpha}\right)}_{F.O.C = 0} + \underbrace{\frac{\partial r}{\partial \alpha} \left(\frac{\partial r}{\partial \alpha} + \frac{\partial r}{\partial \alpha}\right)}_{F.O.C = 0} + \underbrace{\frac{\partial r}{$$

Putting it all together:

$$\frac{\partial T}{\partial \alpha} = -\frac{1}{0 - \left(\frac{\partial L}{\partial T \partial \lambda}\right)^2} \left[-\frac{\partial L}{\partial \lambda \partial T} \cdot \frac{\partial L}{\partial \alpha \partial \lambda} \right]$$

$$= \underbrace{\frac{1}{\left(\frac{\partial L}{\partial T \partial \lambda}\right)^2}}_{+} \left[-\underbrace{\frac{\partial L}{\partial \lambda \partial T}}_{-} \cdot \underbrace{\frac{\partial L}{\partial \alpha \partial \lambda}}_{-} \right]$$

$$< 0$$

We can also apply the IFT to understand how T changes with β .

$$\begin{bmatrix} \frac{\partial T}{\partial \beta} \\ \frac{\partial \lambda}{\partial \beta} \end{bmatrix} = -\begin{bmatrix} \frac{\partial^2 L}{\partial T^2} & \frac{\partial L}{\partial \lambda \partial T} \\ \frac{\partial L}{\partial T \partial \lambda} & \frac{\partial^2 L}{\partial \lambda^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial \beta \partial T} \\ \frac{\partial L}{\partial \beta \partial \lambda} \end{bmatrix}$$

$$= -\frac{1}{\text{Determinant}} \begin{bmatrix} \frac{\partial^2 L}{\partial \lambda^2} & -\frac{\partial L}{\partial \lambda \partial T} \\ -\frac{\partial L}{\partial T \partial \lambda} & \frac{\partial^2 L}{\partial T^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial \beta \partial T} \\ \frac{\partial L}{\partial \beta \partial \lambda} \end{bmatrix}$$

$$= -\frac{1}{\text{Determinant}} \begin{bmatrix} 0 & -\frac{\partial L}{\partial \lambda \partial T} \\ -\frac{\partial L}{\partial T \partial \lambda} & \frac{\partial^2 L}{\partial T^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial \beta \partial T} \\ \frac{\partial L}{\partial \beta \partial \lambda} \end{bmatrix}$$

We compute the only new term required:

$$\frac{\partial L}{\partial \beta \partial \lambda} = \int_{\frac{q^* - e^*}{r^*}}^{\infty} -r f_{\varepsilon}(\varepsilon) d\varepsilon - \frac{\partial}{\beta} \left(\frac{q^* - e^*}{r^*} \right) (q^* - T - \beta r) f_{\varepsilon}(\cdot)$$

$$+ \int_{0}^{\frac{q^* - e^*}{r^*}} -r f_{\varepsilon}(\varepsilon) d\varepsilon + \frac{\partial}{\beta} \left(\frac{q^* - e^*}{r^*} \right) \left(\frac{1}{\alpha} - \beta r \right) f_{\varepsilon}(\cdot)$$

$$= -r$$

Putting it all together:

$$\frac{\partial T}{\partial \beta} = \underbrace{\frac{1}{\left(\frac{\partial L}{\partial T \partial \lambda}\right)^2}}_{+} \left[-\underbrace{\frac{\partial L}{\partial \lambda \partial T}}_{-} \cdot \underbrace{\frac{\partial L}{\partial \beta \partial \lambda}}_{-} \right]$$

Unconstrained case

The owner chooses T to maximize his utility:

$$\max_{T} \qquad T \cdot Pr(q \ge q^*) + E[\tilde{q} \mid q < q^*] \cdot Pr(q < q^*) - \gamma(r)$$

Expressing in terms of exogenous variables:

$$\max_{T} \qquad T \cdot Pr\left(\varepsilon \geq \frac{q^* - e^*}{r}\right) + E\left[e + r\varepsilon - \frac{1}{2\alpha} \mid \varepsilon < \frac{q^* - e^*}{r}\right] \cdot Pr\left(\varepsilon < \frac{q^* - e^*}{r}\right) - \gamma(r)$$

Translating into integral notation:

$$L = \underbrace{T \int_{\frac{q^* - e^*}{r^*}}^{\infty} f_{\varepsilon}(\varepsilon) d\varepsilon}_{A} + \underbrace{\int_{0}^{\frac{q^* - e^*}{r^*}} \left(e + r\varepsilon - \frac{1}{2\alpha}\right) f_{\varepsilon}(\varepsilon) d\varepsilon}_{B} - \gamma(r)$$

Taking the derivative with respect to T

$$=\underbrace{\int_{\frac{q^*-e^*}{r_*}}^{\infty}f_{\varepsilon}(\varepsilon)d\varepsilon}_{A'} + T\underbrace{\left[0 - \frac{\partial}{\partial T}\underbrace{\left(\frac{q^*-e^*}{r_*}\right)f_{\varepsilon}(\cdot)}_{F^*}\right]}_{A'} + \underbrace{\int_{0}^{\frac{q^*-e^*}{r_*}}\left(\frac{\partial e}{\partial T} + \frac{\partial r}{\partial T}\varepsilon\right)f_{\varepsilon}(\varepsilon)d\varepsilon}_{B'} + \underbrace{\int_{0}^{q^*-e^*}\underbrace{\left(\frac{\partial e}{\partial T} + \frac{\partial r}{\partial T}\varepsilon\right)f_{\varepsilon}(\varepsilon)d\varepsilon}_{B'} + \underbrace{\int_{0}^{q^*-e^*}\underbrace{\left(\frac{\partial e}{\partial T} + \frac{\partial r}{\partial T}\varepsilon\right)f_{\varepsilon}(\varepsilon)d\varepsilon}_{B'} - \underbrace{\frac{\partial}{\partial T}\left(\frac{q^*-e^*}{r^*}\right)\left(\frac{1}{\alpha}\right)f_{\varepsilon}(\cdot)}_{B'} - \gamma'(r)\frac{\partial r}{\partial T}}_{B'}$$

$$=\underbrace{\left(1 - F_{\varepsilon}(\cdot)\right)}_{A'} + \underbrace{\frac{\partial e}{\partial T}F_{\varepsilon}(\cdot) + \frac{\partial r}{\partial T}\left(\int_{0}^{\frac{q^*-e^*}{r^*}}\varepsilon f_{\varepsilon}(\varepsilon)d\varepsilon}_{B'}\right) - \frac{1}{\alpha}f_{\varepsilon}(\cdot)\frac{\partial}{\partial T}\left(\frac{q^*-e^*}{r^*}\right)}_{B'} - \gamma'(r)\frac{\partial r}{\partial T}}_{B'}$$

Next we apply the IFT to understand how T changes with α :

$$\frac{\partial T}{\partial \alpha} = -\frac{\frac{\partial L}{\partial \alpha \partial T}}{\frac{\partial^2 L}{\partial T^2}}$$

We know that $\frac{\partial^2 L}{\partial T^2} < 0$ because it's a second order condition. Now let's see if we can express $\frac{\partial L}{\partial \alpha \partial T}$ as a function of the S.O.C

$$\begin{split} &\frac{\partial e}{\partial T} = -\frac{1}{D} \left[\frac{\partial^2 L}{\partial r^2} \cdot \frac{\partial L}{\partial T \partial e} - \frac{\partial L}{\partial r \partial e} \cdot \frac{\partial L}{\partial T \partial r} \right] \\ &\frac{\partial e}{\partial \alpha} = -\frac{1}{D} \left[\frac{\partial^2 L}{\partial r^2} \cdot \frac{\partial L}{\partial \alpha \partial e} - \frac{\partial L}{\partial r \partial e} \cdot \frac{\partial L}{\partial \alpha \partial r} \right] \\ &\frac{\partial r}{\partial T} = -\frac{1}{D} \left[-\frac{\partial L}{\partial e \partial r} \cdot \frac{\partial L}{\partial T \partial e} + \frac{\partial^2 L}{\partial e^2} \cdot \frac{\partial L}{\partial T \partial r} \right] \\ &\frac{\partial r}{\partial \alpha} = -\frac{1}{D} \left[-\frac{\partial L}{\partial e \partial r} \cdot \frac{\partial L}{\partial \alpha \partial e} + \frac{\partial^2 L}{\partial e^2} \cdot \frac{\partial L}{\partial \alpha \partial r} \right] \end{split}$$

Where

$$\frac{\partial L}{\partial T \partial e} = -\frac{1}{r} f_{\varepsilon}(\cdot)$$

$$\frac{\partial L}{\partial \alpha \partial e} = \frac{1}{\alpha^{2} r} f_{\varepsilon}(\cdot)$$

$$\frac{\partial L}{\partial T \partial r} = -\frac{1}{r} f_{\varepsilon}(\cdot) \left(\frac{q^{*} - e}{r}\right)$$

$$\frac{\partial L}{\partial \alpha \partial r} = \frac{1}{\alpha^{2} r} f_{\varepsilon}(\cdot) \left(\frac{q^{*} - e}{r}\right)$$

Therefore we can write:

$$\begin{split} \frac{\partial e}{\partial \alpha} &= -\frac{1}{\alpha^2} \frac{\partial e}{\partial T} \\ \frac{\partial}{\partial \alpha} \left[\frac{\partial e}{\partial T} \right] &= -\frac{1}{\alpha^2} \frac{\partial}{\partial T} \left[\frac{\partial e}{\partial T} \right] \\ \frac{\partial r}{\partial \alpha} &= -\frac{1}{\alpha^2} \frac{\partial r}{\partial T} \\ \frac{\partial}{\partial \alpha} \left[\frac{\partial r}{\partial T} \right] &= -\frac{1}{\alpha^2} \frac{\partial}{\partial T} \left[\frac{\partial e}{\partial T} \right] \end{split}$$

Also as a sanity check:

$$\begin{split} \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) &= \frac{\left(1 - \frac{\partial e}{\partial T} \right) r - \left(q^* - e \right) \frac{\partial r}{\partial T}}{r^2} \\ \frac{\partial}{\partial \alpha} \left(\frac{q^* - e}{r} \right) &= \frac{-\left(\frac{1}{\alpha^2} - \frac{\partial e}{\partial \alpha} \right) r - \left(q^* - e \right) \frac{\partial r}{\partial \alpha}}{r^2} = \frac{-\left(\frac{1}{\alpha^2} + \frac{1}{\alpha^2} \frac{\partial e}{\partial T} \right) r - \left(q^* - e \right) \left(-\frac{1}{\alpha^2} \right) \frac{\partial r}{\partial T}}{r^2} = -\frac{1}{\alpha^2} \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \frac{\partial r}{\partial T} \left(\frac{q^* - e}{r} \right) \frac{\partial r}{\partial$$

Finally:

$$\begin{split} \frac{\partial L}{\partial T^2} &= -f_{\varepsilon}(\cdot) \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) + \frac{\partial^2 e}{\partial T} F_{\varepsilon}(\cdot) + \frac{\partial e}{\partial T} f_{\varepsilon}(\cdot) \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) + \frac{\partial^2 r}{\partial T} \left(\int_0^{\frac{q^* - e^*}{r^*}} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon \right) \\ &+ \frac{\partial r}{\partial T} \left[0 + \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \left(\frac{q^* - e^*}{r^*} \right) f_{\varepsilon}(\cdot) \right] - \frac{1}{\alpha} f_{\varepsilon}'(\cdot) \left[\frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right]^2 - \frac{1}{\alpha} f_{\varepsilon}(\cdot) \frac{\partial}{\partial T} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right] - \frac{\partial}{\partial T} \left[\gamma'(r) \frac{\partial r}{\partial T} \right] \\ &= - \left(1 - \frac{\partial e}{\partial T} \right) f_{\varepsilon}(\cdot) \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) + \frac{\partial^2 e}{\partial T} F_{\varepsilon}(\cdot) + \frac{\partial^2 r}{\partial T^2} \left(\int_0^{\frac{q^* - e^*}{r^*}} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon \right) + \frac{\partial r}{\partial T} f_{\varepsilon}(\cdot) \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \left(\frac{q^* - e^*}{r^*} \right) \right) \\ &- \frac{1}{\alpha} f_{\varepsilon}'(\cdot) \left[\frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right]^2 - \frac{1}{\alpha} f_{\varepsilon}(\cdot) \frac{\partial}{\partial T} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right] - \left[\gamma''(r) \left(\frac{\partial r}{\partial T} \right)^2 + \gamma'(r) \frac{\partial^2 r}{\partial T^2} \right] \end{split}$$

And:

$$\begin{split} \frac{\partial L}{\partial \alpha \partial T} &= -f_{\varepsilon}(\cdot) \frac{\partial}{\partial \alpha} \left(\frac{q^* - e}{r} \right) + \frac{\partial}{\partial \alpha} \left[\frac{\partial e}{\partial T} \right] F_{\varepsilon}(\cdot) + \frac{\partial e}{\partial T} f_{\varepsilon}(\cdot) \frac{\partial}{\partial \alpha} \left(\frac{q^* - e}{r} \right) + \frac{\partial}{\partial \alpha} \left[\frac{\partial r}{\partial T} \right] \left(\int_{0}^{\frac{q^* - e^*}{r^*}} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon \right) \\ &+ \frac{\partial r}{\partial T} \left[0 + \frac{\partial}{\partial \alpha} \left(\frac{q^* - e}{r} \right) \left(\frac{q^* - e^*}{r^*} \right) f_{\varepsilon}(\cdot) \right] - \frac{1}{\alpha} f'_{\varepsilon}(\cdot) \frac{\partial}{\partial \alpha} \left(\frac{q^* - e}{r} \right) \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) - \frac{1}{\alpha} f_{\varepsilon}(\cdot) \frac{\partial}{\partial \alpha} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right] \\ &+ \frac{1}{\alpha^2} f_{\varepsilon} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right] - \frac{\partial}{\partial \alpha} \left[\gamma'(r) \frac{\partial r}{\partial T} \right] \\ &= - \left(1 - \frac{\partial e}{\partial T} \right) f_{\varepsilon}(\cdot) \frac{\partial}{\partial \alpha} \left(\frac{q^* - e}{r} \right) + \frac{\partial}{\partial \alpha} \left[\frac{\partial e}{\partial T} \right] F_{\varepsilon}(\cdot) + \frac{\partial}{\partial \alpha} \left[\frac{\partial r}{\partial T} \right] \left(\int_{0}^{\frac{q^* - e^*}{r^*}} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon \right) + \frac{\partial r}{\partial T} f_{\varepsilon}(\cdot) \frac{\partial}{\partial \alpha} \left(\frac{q^* - e}{r} \right) \left(\frac{q^* - e^*}{r^*} \right) \\ &- \frac{1}{\alpha} f'_{\varepsilon}(\cdot) \frac{\partial}{\partial \alpha} \left(\frac{q^* - e}{r} \right) \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) - \frac{1}{\alpha} f_{\varepsilon}(\cdot) \frac{\partial}{\partial \alpha} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right] \\ &+ \frac{1}{\alpha^2} f_{\varepsilon} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right] - \left[\gamma''(r) \left(\frac{\partial r}{\partial \alpha} \right) \left(\frac{\partial r}{\partial T} \right) + \gamma'(r) \frac{\partial}{\partial \alpha} \frac{\partial r}{\partial T} \right] \\ &= -\frac{1}{\alpha^2} \left[- \left(1 - \frac{\partial e}{\partial T} \right) f_{\varepsilon}(\cdot) \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right] - \frac{1}{\alpha^2} \left[\frac{\partial^2 e}{\partial T^2} \right] F_{\varepsilon}(\cdot) - \frac{1}{\alpha^2} \left[\frac{\partial^2 r}{\partial T^2} \right] \left(\int_{0}^{\frac{q^* - e^*}{r^*}} \varepsilon f_{\varepsilon}(\varepsilon) d\varepsilon \right) \\ &- \frac{1}{\alpha^2} \left[\frac{\partial r}{\partial T} f_{\varepsilon}(\cdot) \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \left(\frac{q^* - e^*}{r^*} \right) \right] - \frac{1}{\alpha^2} \left[- \frac{1}{\alpha} f'_{\varepsilon}(\cdot) \left[\frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right]^2 \right] - \frac{1}{\alpha^2} \left[- \frac{1}{\alpha} f_{\varepsilon}(\cdot) \frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right] \right] \\ &+ \frac{1}{\alpha^2} f_{\varepsilon} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e}{r} \right) \right] + \frac{1}{\alpha^2} \left[\gamma''(r) \left(\frac{\partial r}{\partial T} \right) \left(\frac{\partial r}{\partial T} \right) + \gamma'(r) \frac{\partial}{\partial T} \frac{\partial r}{\partial T} \right] \\ &= - \underbrace{\frac{1}{\alpha^2} \left(\frac{\partial^2 r}{\partial T^2} \right) + \underbrace{\frac{1}{\alpha^2} f_{\varepsilon} \left[\frac{\partial r}{\partial T} \left(\frac{q^* - e}{r} \right) \right]}_{-1} \left[\frac{\partial r}{\partial T} \left(\frac{\partial r}{\partial T} \right) + \gamma'(r) \frac{\partial r}{\partial T} \frac{\partial r}{\partial T} \right]} \\ &= - \underbrace{\frac{1}{\alpha^2} \left(\frac{\partial r}{\partial T} \right) + \underbrace{\frac{1}{\alpha^2} f_{\varepsilon} \left[\frac{\partial r}{\partial T} \left(\frac{q^* - e}{r} \right) \right]}_{-1} \left[\frac{\partial r}{\partial T} \left(\frac{\partial r}{\partial T} \right) + \gamma'(r) \frac{\partial r}{\partial T} \left(\frac{\partial r}{\partial T}$$

Putting it altogether

$$\frac{\partial T}{\partial \alpha} = - \underbrace{\frac{\partial L}{\partial \alpha \partial T}}_{-}$$

> 0

Applying the same analysis to β , is much simpler because $\frac{\partial}{\partial \beta} \frac{\partial r}{\partial T}$ and $\frac{\partial}{\partial \beta} \frac{\partial e}{\partial T}$ equal zero. Moreover:

$$\frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) = \frac{-\frac{\partial e}{\partial \beta} r - (q^* - e^*) \frac{\partial r}{\partial \beta}}{(r^*)^2} < 0$$

$$\frac{\partial}{\partial T} \left(\frac{q^* - e^*}{r^*} \right) = \frac{\left(1 - \frac{\partial e}{\partial T} \right) r - (q^* - e) \frac{\partial r}{\partial T}}{r^2} > 0$$

$$\frac{\partial}{\partial \beta} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e^*}{r^*} \right) \right] = \frac{r^2 \left[\left(1 - \frac{\partial e}{\partial T} \right) \frac{\partial r}{\partial \beta} + \frac{\partial e}{\partial \beta} \frac{\partial r}{\partial T} \right] - 2r \frac{\partial r}{\partial \beta} \left[\left(1 - \frac{\partial e}{\partial T} \right) r - (q^* - e) \frac{\partial r}{\partial T} \right]}{(r^*)^4}$$

$$= \frac{r^2 \left(1 - \frac{\partial e}{\partial T} \right) \frac{\partial r}{\partial \beta} + r^2 \frac{\partial e}{\partial \beta} \frac{\partial r}{\partial T} - 2r^2 \left(1 - \frac{\partial e}{\partial T} \right) \frac{\partial r}{\partial \beta} + 2r (q^* - e) \frac{\partial r}{\partial \beta} \frac{\partial r}{\partial T}}{(r^*)^4}$$

$$= \frac{-r^2 \left(1 - \frac{\partial e}{\partial T} \right) \frac{\partial r}{\partial \beta} + r^2 \frac{\partial e}{\partial \beta} \frac{\partial r}{\partial T} + 2r (q^* - e) \frac{\partial r}{\partial \beta} \frac{\partial r}{\partial T}}{(r^*)^4}$$

$$> 0$$

Then

$$\begin{split} \frac{\partial L}{\partial \beta \partial T} &= -f_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) + \frac{\partial}{\partial \beta} \left[\frac{\partial e}{\partial T} \right] F_{\varepsilon}(\cdot) + \frac{\partial e}{\partial T} f_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left(\frac{q^* - e}{r} \right) + \frac{\partial}{\partial \beta} \left[\frac{\partial r}{\partial T} \right] \int_{-\infty}^{q^* - e^*} f_{\varepsilon}(\cdot) d\varepsilon \\ &+ \frac{\partial r}{\partial T} \left[0 + \frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) \left(\frac{q^* - e^*}{r^*} \right) f_{\varepsilon}(\cdot) \right] - \frac{1}{\alpha} f'_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) \frac{\partial}{\partial T} \left(\frac{q^* - e^*}{r^*} \right) - \frac{1}{\alpha} f_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e^*}{r^*} \right) \right] \\ &- \frac{\partial}{\partial \beta} \left[\gamma'(r) \frac{\partial r}{\partial T} \right] \\ &= -f_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) + \frac{\partial e}{\partial T} f_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left(\frac{q^* - e}{r} \right) \\ &+ \frac{\partial r}{\partial T} \left[\frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) \left(\frac{q^* - e^*}{r^*} \right) f_{\varepsilon}(\cdot) \right] - \frac{1}{\alpha} f'_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) \frac{\partial}{\partial T} \left(\frac{q^* - e^*}{r^*} \right) - \frac{1}{\alpha} f_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e^*}{r^*} \right) \right] \\ &- \left[\gamma''(r) \frac{\partial r}{\partial \beta} \cdot \frac{\partial r}{\partial T} + \gamma'(r) \frac{\partial}{\partial \beta} \frac{\partial r}{\partial T} \right] \\ &+ \frac{1}{2} \left[\frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) + \frac{\partial e}{\partial T} f_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r} \right) + \frac{\partial r}{\partial T} \left[\frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) \left(\frac{q^* - e^*}{r^*} \right) f_{\varepsilon}(\cdot) \right] \\ &- \frac{1}{\alpha} f'_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) \frac{\partial}{\partial T} \left(\frac{q^* - e^*}{r^*} \right) - \left[\gamma''(r) \frac{\partial r}{\partial \beta} \cdot \frac{\partial r}{\partial T} \right] - \frac{1}{\alpha} f_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e^*}{r^*} \right) \right] \\ &+ \frac{\partial r}{\partial T} \left[\frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) \frac{\partial}{\partial T} \left(\frac{q^* - e^*}{r^*} \right) - \left[\gamma''(r) \frac{\partial r}{\partial \beta} \cdot \frac{\partial r}{\partial T} \right] - \frac{1}{\alpha} f_{\varepsilon}(\cdot) \frac{\partial}{\partial \beta} \left[\frac{\partial}{\partial T} \left(\frac{q^* - e^*}{r^*} \right) \right] \\ &+ \frac{\partial r}{\partial T} \left[\frac{\partial}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) \frac{\partial}{\partial T} \left(\frac{q^* - e^*}{r^*} \right) - \left[\gamma''(r) \frac{\partial r}{\partial \beta} \cdot \frac{\partial r}{\partial T} \right] - \frac{\partial r}{\partial T} \left[\frac{\partial r}{\partial \beta} \left(\frac{q^* - e^*}{r^*} \right) \right] \\ &+ \frac{\partial r}{\partial T} \left[\frac{\partial r}{\partial \beta} \left(\frac{\partial r}{\partial \beta} \right) \left(\frac{\partial r}{\partial \beta} \cdot \frac{\partial r}{\partial \beta} \right) \right] \\ &+ \frac{\partial r}{\partial T} \left[\frac{\partial r}{\partial \beta} \left(\frac{\partial r}{\partial \beta} \right) \left(\frac{\partial r}{\partial \beta} \right) \left(\frac{\partial r}{\partial \beta} \right) \right] \\ &+ \frac{\partial r}{\partial T} \left[\frac{\partial r}{\partial \beta} \left(\frac{\partial r}{\partial \beta} \right) \right] \\ &+ \frac{\partial r}{\partial T} \left[\frac{\partial r}{\partial \beta} \left(\frac{\partial r}{\partial \beta} \right) \left(\frac{\partial$$

Putting it altogether

$$\frac{\partial T}{\partial \beta} = -\frac{\overbrace{\frac{\partial L}{\partial \beta \partial T}}^{+}}{\underbrace{\frac{\partial^{2} L}{\partial T^{2}}}_{-}}$$