State-Conditioned Soil Investment in Rural Uganda

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Abstract

While poor soils limit agricultural production across rural, sub-Saharan Africa, most smallholder farmers fail to invest in their soils in the way that soil scientists and policy makers prescribe. A small but growing literature examines biophysical constraints on soil investment, and in particular state-conditioned soil investment — the manner in which current soil fertility drives investment in soils in poor, agricultural contexts. While some research finds that farmers invest more in low fertility soils, other authors find the opposite. We model two types of state-conditioned soil investment, and show that while organic amendments in the form of manure or compost are optimally applied on low fertility plots, structural investments to halt soil degradation may not be optimal on any plots, or may be optimal only on high value, high fertility plots. Using plot-level panel data from Uganda, we find that soil fertility measures from 2003 do predict subsequent soil management practices a decade later. Farmers are more likely to apply organic amendments to low fertility plots, as predicted by our analytical model. Laborious conservation practices and structural investments in plots are quite rare, indicating that these measure may not be effective enough to be profitable in Uganda. Even so, certain conservation practices are predicted by 2003 soil fertility conditions. These associations, and the associations regarding organic amendment, are highly stable in the face of many controls — it appears that soils starting conditions do matter for subsequent soil investment decisions and soil fertility trajectories. Together, the model and empirical investigation bring nuance to the previous discussion on state-conditioned soil investment, and help to resolve seemingly inconsistent empirical findings in the literature.

Keywords: soil fertility, soil investment, starting conditions, mixed integer programming

1 Introduction

Declining soil fertility is frequently cited as the primary biophysical reason for stagnant per capita food production in sub-Saharan Africa (Morris, 2007; Place et al., 2003; Sanchez et al., 1996), and as a potential mechanism for trapping rural, semi-subsistence agricultural communities in poverty (Dasgupta and Mäler, 1990; Barrett and Bevis, 2015). Increases in soil organic carbon, a common measure of soil fertility, have the capacity to sharply increase yields; Lal (2006) estimates that for every ton/hectare increase in soil organic carbon, maize, wheat and rice yields increase by 30-300, 20-70, and 10-50 kg/hectare, respectively, across the developing world. Yet farmers in poor, semi-subsistence settings tend to under-invest in soil conservation measures and apply less than optimal levels of organic and inorganic fertilizer (De Graaff et al., 2008; Hagos and Holden, 2006).

A small body of literature examines the determinants soil investment.[1] Extension contacts and education, for instance, are associated with increased adoption of soil conservation practices in Tanzania (De Graaff et al., 2008), and larger farms adopt these measures more intensively in Peru and Bolivia (Posthumus, 2005) and in Ethiopia (Amsalu and De Graaff, 2007). In the Philippines, farmers are more likely to adopt costly soil conservation practices if crop intensification or cash cropping was possible (Lapar and Pandey, 1999). In India, households with more adult males, more farm servants, and less land invest more in soil conservation (Pender and Kerr, 1998). Plot level factors impact soil conservation practices too — for example, farmers are less likely to adopt conservation measures on plots farther from households in Ethiopia, maybe due to increased labor requirements or less security of tenure (Bekele and Drake, 2003).

Additionally, an even smaller group of papers find soil characteristics themselves (e.g., fertility, erosion, color, perceived quality) to be associated with soil investment (Antle, Stoorvogel and Valdivia, 2006; Amsalu and De Graaff, 2007; Marenya and Barrett, 2009*a*; Yirga and Hassan, 2014). If soil investment is truly state-conditioned in this way, the relationship has important implications for long-term fertility dynamics. If a farmer is most likely to invest, or if he/she invests most, in low fertility plots, while neglecting to invest in higher fertility plots, then we might expect a long-run regression to farm-mean soil fertility. However, if farmers only find it profitable to invest in higher fertility plots, neglecting lower fertility plots as a lost cause, then we might expect divergent trends — degradation for low fertility plots and rehabilitation for high fertility plots.

Yet both theoretical and empirical evidence is mixed with regards to how current or "starting" soil fertility impacts soil investment decisions. Some researchers find that higher soil fertility boosts yield response to compost, mulch or fertilizers, making farmers more likely to invest in their highest fertility soils (Zingore et al., 2008*a*; Bayala et al., 2012; Vanlauwe, Tittonell and Mukalama, 2006; Antle, Stoorvogel and Valdivia, 2006; Stephens et al., 2012; Marenya and Barrett, 2009*b*,*a*; Jama, Buresh and Place, 1998). Other researchers find that farmers are more likely to build conservation structures or invest in long-term conservation practices on poor soils than on fertile soils (Amsalu and De Graaff, 2007; Yirga and Hassan, 2014).

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[1] An extensive body of literature examines adoption of fertilizer, but fertilizer is largely an investment in current period yields, rather than an investment in long-term soil fertility.

This diversity of conclusions is partially explained by the research context from which these papers spring. Most papers on state-conditioned soil investment stem from one of four threads of the technology adoption literature: (i) adoption of soil and water conservation, (ii) adoption of conservation agriculture, (iii) adoption of fertilizer, and (iv) adoption of organic amendments. The first two bodies of literature tend to focus on arid or semi-arid, erosion-prone regions, where soil investment is meant to mitigate topsoil loss. The second two bodies of literature focus more generally on smallholder agriculture in the tropics, and soil investment is meant to increase soil organic carbon or nutrient stocks, and sometimes to increase agricultural productivity. Thus, the relevant dimension of soil fertility and also the relevant measures of soil investment differ by context/literature.

Soil and water conservation (SWC) is an umbrella term for the construction and maintenance of plot-level structures that halt or mitigate soil erosion: primarily terracing, ridging, soil bunds, strip cropping, agro-forestry, and drainage ditches. These practices are most often promoted in hilly areas that are likely to suffer from erosion. Therefore, much of the research on SWC is in mountainous regions — e.g., the highlands of Ethiopia (Kassie et al., 2008; Shiferaw and Holden, 1998), the mountains of Tanzania (Mbaga-Semgalawe and Folmer, 2000), or the Andes (Aad, 2006; Antle, Stoorvogel and Valdivia, 2006). In such contexts, it seems that SWC is most often adopted on soils that are sloped or perceived as eroding (Amsalu and De Graaff, 2007; Bekele and Drake, 2003; Yirga and Hassan, 2014). Additionally, Gebremedhin and Swinton (2003) suggest that SWC is most valuable and most likely to be adopted on soils that are potentially quite fertile but also highly erodible, in areas with good rainfall and therefore high potential productivity.

Conservation agriculture (CA) is an umbrella term for agricultural practices that similarly seek to mitigate soil erosion and nutrient leakage: primarily cover cropping, no till farming, intercropping, fallowing, rotation and mulching. Crop and soil scientists generally find that the yield response to CA is highest on lower fertility soils (Bayala et al., 2012; Buerkert and Stern, 1995; Buerkert, Stern and Marschner, 1995; Buerkert, Bationo and Dossa, 2000; Sileshi et al., 2010). In a review of literature on CA adoption, Knowler and Bradshaw (2007) find that across a number of studies, farmers seem more likely to adopt CA on "highly erodible land" and less likely to adopt CA on "high-productivity" soils. This is in line with the marginal returns documented by soil scientists.

Fertilizer refers to inorganic fertilizer, often containing only nitrogen, potassium and phosphorous but sometimes including an additional mix of trace minerals. A large literature exists on the determinants of fertilizer adoption, but only two papers examine the effect of soil fertility on fertilizer adoption. Marenya and Barrett (2009*b*) find that the yield returns to fertilizer increase with soil organic carbon, and Marenya and Barrett (2009*a*) find that farmers in Kenya are more likely to adopt fertilizer on plots with higher soil organic carbon. This is, again, in line with crop and soil science research showing that better soil structure, higher levels of soil organic carbon, and increased levels of secondary micronutrients all boost the yield response to fertilizer (Kihara et al., 2016; Zingore et al., 2008*b*; Vanlauwe, Tittonell and Mukalama, 2006).

Organic amendments include any amendments that increase soil organic carbon: most often compost, crop residue, food residue, and manure. There is little evidence on the determinants of organic amendment adoption — in part because household-level surveys, even those focused on agriculture, often neglect to gather information on such amendments. However, research by Duncan et al. (2016) and Dawe et al. (2003) suggest that organic amendments contribute most to productivity on very low fertility soils, deficient in soil organic carbon. Thus, it seems likely that in many tropical contexts at least, yield response to organic amendment is highest on low fertility soils, while yield response to fertilizer is highest on high fertility soils.

Drawing key insights from each of these bodies of literature, we build a multi-dimentional model of state-conditioned soil investment. In our model, farmers may invest labor into three categories of activities: (i) activities that increase next period's crop production but have no effect on soil fertility or soil degradation, (ii) activities that increase next period's soil fertility but have no effect on soil degradation, and (iii) activities that slow soil degradation but have no direct impact on soil fertility or crop production. (We define soil degradation as any process leading to lower future soil fertility — often soil erosion or nutrient leaching, in practice.) This is, of course, a stylized version of the wide array of soil investment tactics that a farmer may employ. A farmer might, for instance, engage in practices that both slightly increase next period's soil fertility and also halt ongoing soil degradation. For simplicity however, we allow each form of labor allocation to have only one impact: on crop production, on soil fertility, or on soil degradation. We also discuss implications for agricultural practices that impact multiple outcomes at once.

Our model provides a few key insights. First, investments that simply increase next period's soil fertility, such as organic amendments, are most productively used on lower fertility soils. For a farmer with multiple plots, it will always be optimal to supply the most organic amendment to the lowest fertility plot, and the least organic amendment to the highest fertility plots. Additionally, if investments that halt soil degradation, such as terraces, soil bunds or fences are quite time consuming to maintain, it will rarely be optimal to invest in this way. Similarly, such structural investments will not be optimal on plots with low inherent soil fertility, or for plots that are not experiencing major degradation. However, these structural investments will be optimal on inherently fertile plots suffering from severe degradation, particularly if they have not yet degraded too far and can therefore be rehabilitated quickly.

After discussing these insights, we examine empirical evidence on soil investment from rural Uganda. Using a unique, decade-long plot-level panel dataset we examine multiple dimensions of state-conditioned soil investment by regressing 2013 soil investment practices on 2003 soil fertility measures. The associations estimated cannot be assumed causal, as autocorrelation across time and relevant, omitted variables may bias the estimated coefficients on 2003 plot characteristics. However, we control for a long list of 2013 plot characteristics, including 2013 soil fertility measures, and show that the relationships between 2003 plot conditions and 2013 soil investment measures are stable in the face of these controls. It seems quite likely, therefore, that the relationships estimated are at least partially causal, implying that starting conditions do matter for subsequent soil investment strategies.

Specifically, we find that plots with poor soil fertility in 2003 are more likely to receive organic amendments a decade later in 2013, as our model suggests. Very few plots experience more structural forms of investment, indicating that either such structures are very costly, or they are ineffective, or soil degradation is not a serious problem in Uganda. Even though few farmers invest in conservation practices or structures, it does appear that starting soil conditions are relevant to this choice. Plots that start with higher soil organic carbon and lower macronutrient content are more likely to experience conservation practices.

The next section of this paper lays out our dynamic model for soil investment decisions, and Section 3 discusses our panel data from rural Uganda. Section 4 presents results, both from the dynamic model and from empirical analysis. Section 5 concludes.

2 Model

Assume farming families maximize plot-level profits, for multiple plots per household, over many generations. Time begins at $t = 0$ and goes on infinitely, with profits in time period t discounted by ρ^t, $\rho = (\frac{1}{1+\delta})$, where $\delta > 0$ is the period discount factor. Plot-level profits in each time period t for each plot p are given by price P multiplied by quantity of crop produced G. Because most Ugandan plots contain multiple crops, we interpret G as the joint quantity of all crops produced, and P as the average price per unit of these crops, net of average per unit costs.

As shown in Equation 1, the crop production function takes three arguments: labor devoted directly to crops LC_{pt}, labor allocated towards organic matter amendment LO_{pt}, and soil fertility F_{pt}. Labor directly to crops might take the form of planting, weeding, or pruning. Organic matter amendment might be the application of manure, compost, crop residue, food waste, or other biomass. Soil fertility can be proxied by soil carbon (Lal, 2006; Blanco-Canqui et al., 2013).

The state equation is given by Equation 2. Next period's soil fertility $F_{p,t+1}$ is a product of current soil fertility F_{pt}, labor devoted to organic matter amendment LO_{p0}, and a plot-specific soil degradation rate α_p. The degradation rate α_p is a function of plot-specific structural investments LS_{pt}. It might, in practice, be soil erosion and/or nutrient leakage.

Farmers choose the quantity of organic matter amendment applied to each plot p in every time period t, and hence the proportion of labor devoted to organic amendment, LO_{pt}. This amendment achieves two ends: it increases production in time period t (hence its appearance in Equation 1), and it increases soil fertility in the next time period, $t + 1$ (hence its appearance in Equation 2). This simply reflects the fact that organic amendment has positive yield returns for the majority of soils (Rowe et al. 2006), as well as building up soil organic carbon, a primary component of soil fertility.

We additionally allow farmers to engage in activities that do not build soil fertility directly, but slow or halt soil degradation — i.e., slow the rate at which soils naturally degrade over time, given in this model by α_p. This category of activities reflects physical structures that halt soil erosion and nutrient leaching, e.g. terracing or soil bunds.[2] Unlike organic matter amendment, structural investments in soil fertility on plot p are chosen only once, in $t = 0$. Once chosen, a fixed quantity of labor, LS_{0p}, is devoted to the upkeep of these structures on plot p in every time period t from there on out. That is, $LS_{tp} = LS_{0p} \forall t$. As a result, the plot-specific rate of soil degradation is also set in $t = 0$.

Also unlike organic matter amendment, structural investments do nothing to increase current production, and have no direct effect on next period's soil fertility. (Indirectly, labor allocated towards structural investments actually decreases current production levels by necessarily reducing the labor that can be allocated to crops and organic amendment. For the same reason it also decreases direct investment in soil fertility.)

Farmers manage multiple plots, each beginning with different starting fertility as in Equation 3. Labor is therefore constrained as in Equation 4, where L_t represents total family labor available, and this labor must be allocated amongst all existing plots, and between three tasks per plot. (We assume that farmers employ only family labor — generally true in Uganda.)

$$\max_{LC_{pt}, LO_{pt}, LS_{p0}, F_{pt}} \pi = \sum_p \sum_{t=0}^{t=\infty} \rho^t \{P * G(LC_{pt}, LO_{pt}; F_{pt})\} \tag{1}$$

$$s.t. \;\; F_{p,t+1} = F\big(F_{pt}, LO_{p0}, \alpha_p(LS_{p0})\big) \tag{2}$$

$$F_0 = \big\{F0_1, F0_2, ..., F0_P\big\} \tag{3}$$

$$L_t = \sum_p \Big\{LC_{pt} + LO_{pt} + LS_{p0}\Big\} \tag{4}$$

Equation 5 specifies the production function G as a simple Cobb-Douglas production function with constant returns to scale: $\omega > 0$, $0 < \gamma i < 1$ for each $i \in \{c, s, f\}$, and $\gamma c + \gamma s + \gamma f = 1$. While we realize that assuming this functional form imposes strong restrictions almost certainly not true in reality, it allows for a tractable theoretical model and for easily interpretable relationships between inputs and production. This, in turn, allows us to focus on a more nuanced form for the soil updating function F. Additionally, we impose no restrictions when estimating our empirical relationships.

$$G(LC_{pt}, LO_{pt}; F_{pt}) = \omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_s} F_{pt}^{\gamma_f} \tag{5}$$

The form of the soil updating function F must reflect particular biophysical realities. Soils have a natural upper limit to fertility, an asymptote with respect to soil investment — at some point, additional labor will only maintain soil fertility, but can no longer increase it. Labor allocated towards organic matter amendment must therefore experience diminishing returns: $\frac{dF_{p,t+1}}{dLO_{pt}} > 0$, $\frac{d^2F_{p,t+1}}{dLO_{pt}^2} < 0$. Returns to organic matter amendment must also diminish as soils improve: $\frac{d^2F_{p,t+1}}{dLO_{pt}dF_{pt}} < 0$.

Structural investment affects the state equation in a different way — by reducing soil degradation and therefore increasing the potential steady state for future soil fertility.

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[2] While other practices such as cover cropping might increase soil fertility while also halting soil degradation, in this stylized model we consider only those activities that impact one outcome at a time.

Mindful of these requirements, Equations 6 and 7 define next period's soil fertility F_{t+1} as an exponential function of labor allocated towards organic amendment in period t. Soil fertility is bounded between 0 and 1, an index that could be applied to multiple fertility measures and/or contexts.[3] The exponential form of Equation 7 ensures diminishing returns to labor towards organic amendment. The parameter ζ_{pt} ensures that returns to labor decline in current soil fertility adjusted for degradation, $(F_{pt} - \alpha_p)$.

Equation 8 defines the plot-specific degradation rate α_p as a function of the structural investment decision made in $t = 0$, the plot-specific high degradation rate α_p^H, and the plot-specific low degradation rate α_p^L, as defined in Equation 8. If $\alpha_p^H = \alpha_p^L$, structural investments do nothing to halt degradation on plot p. If $\alpha_p^H > \alpha_p^L$, structural investment slows/halts/reverses degradation and increases the upper asymptote on potential fertility. Soil degradation rates α_p^H and α_p^L are plot-specific as they differ by soil type, slope, climate, and many other plot-specific characteristics.

$$\zeta_{pt} = ln\big(1 - (F_{pt} - \alpha_p)\big) \tag{6}$$

$$F_{p,t+1} = 1 - exp\Bigg\{\zeta_{pt} - \frac{LO_{pt}}{\lambda}\Bigg\} \tag{7}$$

$$\alpha_p = \alpha_p^H - \mathbb{1}\{LS_{p0} > 0\}\Bigg(\alpha_p^H - \alpha_p^L\Bigg) \tag{8}$$

Model parameters are given in Table 1. All parameters are fixed except for the "low" soil degradation rate α_L, for which we consider multiple values, and except for the labor allocation requirement that we assume necessary for structural investment — or the structural labor allocation (SLA). We consider two requirements, SLA = 0.25, and SLA = 0.15. If SLA = 0.25, maintaining structural investments on a plot requires a quarter of the farmer's total available labor. Maintaining structures on 3 plots therefore leaves only a quarter of total labor left for other activities. When SLA = 0.15, maintaining structural investments on a plot requires only fifteen percent of the farmer's total available labor.

To lend intuition to the relationships in the model, Figure 1 illustrates the impacts of labor towards organic amendment and/or structural investment for three theoretical plots, beginning with starting soil fertility at 0.2, 0.5, and 0.8 respectively. For the moment we ignore labor directly to crops, as it has no impact on soil fertility, and imagine time-constant investment strategies where labor is either allocated solely to organic amendment, solely to structural investment, or to both organic amendment and structural investment. In all scenarios, each form of investment is equally divided across the three plots.

In sub-figure [A] at the top of left of Figure 1, we show the impact of allocating all labor into organic amendment, with no labor towards structural investment. Because marginal returns to organic amendment are highest on low fertility plots, and lowest on

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[3] For instance, to transform this index into numbers that would be meaningful for soil carbon content in Uganda, the fertility measure that we use in our empirical analysis, one would divide by 10 such that the upper bound on fertility is 0.10 rather than 1. But in a forested context soil carbon content would have a much higher upper threshold. One might also transform the index to represent soil nitrogen, soil potassium, or cation exchange capacity.

high fertility plots, the fertility of all three plots converges when labor is equally allocated to each plot. The asymptote of convergence is less than one, however, because with no structural investment, soil degradation is occurring at the rate $\alpha_H > 0$.

In sub-figures [B] and [C] we show the opposite: the impact of structural investments built on each plot, but no labor towards organic amendment. In sub-figure [B] α_L is two-thirds the value of α_H, and in sub-figure [C] α_L is only one-third the value of α_H. In both cases, with no organic amendment to build soil fertility over time and with α_L small but still greater than 0, soils degrade over time. However, in sub-figure [C] the process of degradation takes longer, because the α_L of sub-figure [C] is lower than the α_L of sub-figure [D].

In sub-figures [D], [E] and [F] we show the impact of allocating labor towards both structural investment and organic amendment. For these figures, we assume that structural investment takes twenty-five percent of total labor supply per plot, leaving only a quarter of total labor supply available for organic amendment. In sub-figure [D], with α_L two-thirds the value of α_H, soils converge to a much lower stable state than they did under scenario [A] with organic amendment only — the low degradation vs. high degradation trade-off does not compensate for the lack of organic amendment. In sub-figure [E] we assume that α_L is one-third the value of α_H. This makes structural investment more effective at halting soil degradation, but soil still converge to a (slightly) lower stable state than they did under scenario [A]. In sub-figure [F] we assume that $\alpha_L = 0$ — that is, soil investment completely halts degradation. Under this scenario, allocating labor towards both structural investments and organic amendment actually results in a better state than allocating labor only towards organic amendment.

The bottom panel of Figure 1 replicates the middle panel of Figure 1, but we assume that structural investment for each plot takes only fifteen percent of total labor supply, leaving slightly over half of total labor supply available for organic amendment. With structural investment less laborious, it pays off even when it is less effective. Comparing sub-figures [A] and [E] shows that "more laborious" structural investments lead to lower soil fertility levels even when α_L is one-third the value of α_H; comparing sub-figures [A] and [H] shows that "less laborious" structural investments lead to higher soil fertility levels when α_L is one-third the value of α_H. Even when α_L is two-thirds the value of α_H, as in sub-figure [G], structural investments are almost (but not quite) worth the labor loss from organic amendments.

The soil fertility trends of Figure 1 are not stemming from "optimal" behavior — they are simply the outcomes associated with a few simplified, constant soil investment strategies. The key insights to be gained from this figure are, (i) while structural investments halt soil degradation, they cannot build soil fertility unless accompanied by organic amendment, (ii) if structural investments are laborious, or not very effective at halting soil degradation rates, they will not be worth the loss of labor towards organic amendment, (iii) when structural investments are less laborious and also effective at slowing soil degradation, they will lead to higher soil fertility in the long run, even while necessitating less labor allocation towards organic amendment.

If the farmer's objective was to maximize soil fertility over time, it seems that labor allocation towards in structural investments would depend only on the labor intensity of structural investment and the values of α_L vs. α_H. However, the farmer's objective is to maximize production, not soil fertility; soil fertility is merely an input to production. Labor allocated towards organic amendment detracts from labor allocated directly to production, LC_{pt}. Labor allocated towards structural investments detracts from both labor to organic amendments and also labor directly to production.

We therefore solve numerically for optimal labor allocations according to the full model laid out in Equations 1 - 7. We do this by assuming that a steady state exists, after which both labor allocations and soil fertility become constant over time. Appendix 1 derives Equation 9, the analytical, plot-specific condition that defines such an optimal steady state.[4] The left hand side of this equation should converge towards zero as our numerically derived labor allocations and soil fertility asymptote.

$$\frac{F_{pt}}{\gamma_f}\left(\frac{\gamma_c}{LC_{pt}} - \frac{\gamma_o}{LO_{pt}}\right) - \left(\frac{1}{\rho\lambda}\right)\left[\frac{1 - F_{pt}}{\delta + 1 - e^{-\frac{LO_{pt}}{\lambda}}}\right] = 0 \tag{9}$$

Building on this analytically derived steady state, Appendix 2 restates the optimization problem in a form that can be solved numerically to find optimal labor and soil fertility trajectories — assuming that these trajectories evolve only between $t \in \{1, T-1\}$, but that a steady state is reached by period T. This assumption is key, as it allows for us to find optimal labor and soil fertility trajectories over a finite time period, but without limiting the farmer's objective function to only this time period.

Because labor towards structural investment is discrete choice and labor towards organic amendment is a continuous allocation, this is a mixed integer programming model. We optimize over the continuous choice variable for every possible combination of discrete choice variables, and then choose the discrete choice variable combination that best maximizes expected profit. Appendix 3 holds details on computational methods.

3 Data

For empirical analysis, we use a unique, plot-level panel dataset from rural Uganda. The first wave of data was collected during the summer of 2003, by the International Food Policy Research Institute (IFPRI). This IFPRI survey was run in conjunction with a larger Uganda Bureau of Statistics (UBOS) survey conducted in 2002/2003. Together, the surveys collected household-level socioeconomic data, plot-level input and production data, and took plot-level soil samples for later soil analysis. Information on the sampling strategy used in 2003 can be found in Nkonya et al. (2008).[5]

The second wave of data was collected during the summer of 2013 under a National Science Foundation (NSF) funded project. The same household- and plot-level data were collected, along with plot-level soil samples. Of the 859 households interviewed in 2003, 803 were tracked successfully and re-interviewed. Additionally, individuals who

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[4] Additionally, the labor constraint must be satisfied, but this trivially true, as the labor constraint must be satisfied in every time period.

[5] Essentially, rural households were randomly chosen within survey districts, but the survey districts themselves were chosen to represent various agro-ecological zones across Uganda. Thus, the results in this paper cannot be viewed as nationally representative.

had split off from the original 2003 household to form a new household were tracked if still residing within the original parish.[6]

In each survey wave, soil samples were aggregated from 12-20 subsamples (based on plot size) taken in a zig-zag pattern across each plot. Samples were then analyzed for a number of biophysical and chemical characteristics at the National Agricultural Research Laboratory in Uganda using well-established protocols. More details on soil sampling as well as soil analysis can be found in Appendix 4.

In this paper, the unit of analysis is a single plot of land, used to grow a single crop or multiple, mixed crops. Most, though not all, farmers have multiple plots in both 2003 and 2013. While the precise size and shape of these plots tends to shift slightly across the decade, many of them are generally in the same location within a larger parcel. Because GPS waypoints were taken around the corners of all plots in both rounds of data collection, we can match plots across time using their geospatial location. This is how we form a plot-level panel dataset. Of course, some plots cannot be matched over time, as they have no geospatial overlap with another plot from across the decade. These plots are dropped from our analysis. In other cases, a plot from 2003 may have split into two or more plots by 2013. In such cases the 2003 plot is matched with both of the 2013 plots.[7]

Tables 2 and 3 summarize all variables used for analysis. Table 2 reports means (i.e., proportions) for the binary variables that represent investment in soils. Column 1 holds means across plots, and column 2 holds means across houses. Investment in organic amendment is clearly quite low — only 10 percent of plots receive any type of organic amendment, and only 15 percent of households report applying any type of organic amendment. Conservation practices and conservation structures are also rare, with the exception of crop rotation. Forty percent of plots experience rotation, while only 9 and 14 percent of plots receive fallowing and mulching, respectively, and only 3 percent of plots receive cover cropping or no till agriculture. Only 6 percent of plots hold ridging or strips meant to mitigate erosion, only 9 percent plots have terracing, and only 2 percent plots hold drainage ditches. Only 16 percent of plots hold even one type of conservation structure.

Table 3 describes variables that we use to predict soil investment practices. At the top are the three key predictors of interest — soil organic carbon and soil macronutrients in 2003, and plot value in 2003. Soil organic carbon and soil macronutrients both capture a key dimension of starting soil fertility. Plot value, measured in 2003 revenue per hectare, captures the overall productivity potential of a plot. While some of this potential will stem from soil fertility, it will also stem from other, unobserved plot characteristics such as slope, position on the landscape, proximity to pathways, water-holding capacity, etc.

Below those key variables in Table 3 are the 2013 plot characteristics relevant to 2013

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[6] Households that attrited in round two were more likely live in peri-urban areas, slightly younger, smaller, and more educated, with slightly less land and fewer animals.

[7] Of the 1,089 plots from 2003 that were successfully matched to a plot from 2013, 69 percent were matched to exactly 1 plot from 2013, 19 percent were matched to 2 plots from 2013, and the rest were matched to 3-9 plots from 2013. Of the 2003 plots that do split into multiple 2013 plots, 13 percent were under new household ownership in 2013 (usually because they were inherited by a child). The rest were split into multiple plots still owned by the same household.

soil investment. We control for these variables in order to gauge the stability of the relationship between 2003 soil fertility characteristics and 2013 soil investment practices. Most plots are owned and managed by the household head. Crop dummies denote that a plot holds a particular type crop — tubers, cereals, legumes, bananas, cash crops — but not that it only grows that type of crop. Plots are generally quite small — while the average plot size is 0.32 hectares, the median plot size is only 0.19 hectares. The final set of controls includes soil organic carbon in 2013, soil macronutrients in 2013, and plot productivity in 2013. While soil organic carbon has risen slightly over the decade, the difference is not significant when log variables are subjected to a t-test of means. The soil macronutrient variable differs across years, as total K was not collected in 2013. Productivity, measured in revenue per hectare, has risen significantly over the decade.

4 Results

Model Simulations

Setting parameters as in Table 1, we run the maximization problem over 250 time periods for a household with three plots. (More details on the maximization process and computational details in Appendix 2 and Appendix 3.) Starting soil fertility for those plots is distributed as in the theoretical scenarios discussed previously: 0.2, 0.5, and 0.8. As assumed in the middle panel of Figure 1, we initially set the labor allocation towards structural investment on any plot to be one quarter of total labor allocation (SLA = 0.25), and $\alpha_L = (2/3)\alpha_H$.

The results are given in Figures 2 and 3. With structural investment being fairly laborious, and not very effective at halting soil degradation, no labor is allocated towards structural investments. Instead, labor is allocated to organic amendment and directly to crops. The plot that begins with lowest soil fertility (and therefore highest marginal returns to organic amendment) initially receives the most labor towards organic amendment, and the plot that begins with highest soil fertility initially receives the least.

A steady state is reached fairly quickly. Around $t = 20$ the soil fertility levels of all plots converge and asymptote. Around $t = 50$ the labor trajectories also converge and asymptote. Also starting around this point the steady state condition is met — perfectly for one plot, and imperfectly for two plots, for which the steady state condition hovers around zero. This check confirms that the steady state reached numerically is, indeed, the steady state found analytically in Appendix 1.

The same results are found if we allow structural investment to be much more effective, e.g., $\alpha_L = (1/3)\alpha_H$, or even if we allow structural investment to completely halt degradation, i.e., $\alpha_L = 0$. If structural investment requires a quarter of labor to be invested per plot (SLA = 0.25), it is never optimal for the farmer to invest in such structures. The trade-off with crop production is too high.

We next allow structural soil investment to require less labor — only fifteen percent of total labor per plot, as assumed in the bottom panel of Figure 1. We do this for three scenarios of α_L values: $\alpha_L = (1/5)\alpha_H$, $\alpha_L = (1/6)\alpha_H$, and $\alpha_L = (1/10)\alpha_H$. If $\alpha_L > (1/5)\alpha_H$, then structural investment is not optimal, and no plots receive labor allocation towards structural investments. Results will again be identical to those in Figures 2 and 3.

However, once $\alpha_L = (1/5)\alpha_H$, structural investment will be optimal on the highest fertility plot only. The top two trajectories of Figure 4 show the soil fertility and steady state conditions resulting from this maximization. (Profit trajectories can be found in Appendix 5.) In this scenario, structural investments have been made in the highest fertility plot, and therefore its ongoing soil fertility trajectory is raised significantly above the others. For the two lower fertility plots, the return on structural investments is simply too far in the future to be worthwhile.

If α_L lowers even further, to reach $\alpha_L = (1/6)\alpha_H$, structural investment will be optimal on the highest fertility plot and the medium fertility plot, as pictured in the middle two trajectories of Figure 4. In this scenario, the soil fertility trajectory of two plots has raised, and only the lowest-starting fertility plot receives no investment.

Once $\alpha_L \leq (1/10)\alpha_H$, structural investment will be optimal on all plots, as pictured in the bottom two trajectories of Figure 4. In this last scenario, the soil fertility trajectories will converge once more, to a higher asymptote than possible without structural investments. In all of these scenarios a steady state is reached quickly, and the steady state condition is met. Farm-level profits are higher, however, in scenarios with more structural investment and therefore higher farm-average soil fertility levels. Full optimization results for all three scenarios, including profit trajectories, can be found in Appendix 5.

We initially wondered about differential investment patterns between plots starting with low soil fertility vs. plots starting with high soil fertility. Figures 1 and 2 indicate that short-term soil investment is primarily profitable on the lower fertility plots. In panel data, we are therefore likely to observe short-term investment on plots that were previously measured as having lower soil fertility. Plots that were previously measured as having higher soil fertility will be less likely to receive such investment. While structural investment appears to be unprofitable under most conditions and on most plots, it is profitable under particular situations — namely, when the cost of investment is low, and/or when degradation is largely mitigated by such structures. It is also most likely to be profitable on the higher fertility soils, which will most quickly provide a return on such investment.

These insights mirrors empirical findings by De Graaff et al. (2008), Hagos and Holden (2006) and others, who find that farmer rationally fail to invest in long-term soil maintenance practices. They also mirror findings by Gebrehehid & Swinton (2003), who suggest that soil conservation structures are most valuable on highly erodible soils with high potential fertility.

Note that these results are not affected by the basis functions used to reduce dimensionality in the labor allocation trajectories. The results displayed in Figure 4 and Appendix 5 result from using the exponential series as a finite approximation to continuous labor trajectories. Optimization is done over the coefficients on this series. However, the same results will be found if Legendre polynomials are used to approximate continuous labor trajectories. These results are found in Appendix 6.

More details on computation is found in Appendix 3.

Econometric Results

While our model separately examines (i) soil investment that builds soil fertility but does not slow soil degradation, and (ii) soil investment that does not build soil fertility but slows the degradation process, observed practices are less cleanly categorized. We first examine organic amendment, which truly fits the first category of investment. We then examine several practices and structures that roughly fit the second category of investment, but may also contribute directly to soil fertility or even crop production.

Table 2 highlights how rare soil investment is. Only ten percent of plots and fifteen percent of households record any form of organic amendment. And while 58 percent of plots experience some sort of conservation practice meant to slow soil degradation, most of these plots are under rotation — only 18 percent of plots experience a non-rotation conservation practice. Similarly, only 16 percent of plots hold a physical conservation structure — ridging, terraces or drainage ditches.

Our model suggests that the rarity of conservation practices and conservation structures points to either (i) these practices/structures being very costly to maintain, and/or (ii) these practices/structures being fairly ineffective when it comes to halting soil degradation. It might also be that in this context, rapid soil degradation is simply not occurring.[8] The rarity of organic amendment is more puzzling, and may point to a competition for organic resources unaccounted for in our model (Berazneva et al., 2016).

Using plot-level panel data from Uganda, we examine the relationship between 2003 soil fertility indicators and 2013 soil investment practices. We use both 2003 soil carbon content and 2003 soil macronutrient content (sum nitrogen, phosphorus, and potassium) to proxy for "soil starting fertility." We use 2003 land productivity, measured in terms of revenue per hectare, to proxy for the "value" of the plot to the farmer. We do this both for the individual practices listed in Table 2, as well as for summary indicators for any organic amendment being used, any conservation practices being used, and any conservation structures being present. All regressions contain household fixed effects, to mirror the within-household decisions process of our model.

Table 4 examines the 2003 plot conditions associated with 2013 organic amendments. The application of mulch and crop residue (columns 1 and 2) are significantly, negatively associated with 2003 soil organic carbon. This suggests that, as in our model, farmers apply greater levels of mulch and crop residue to lower fertility plots, and lower levels to higher fertility plots. Neither the application of compost nor manure (columns 3 and 4) is significantly associated with 2003 soil organic carbon. When a binary variable is created to indicate any organic amendment being applied (column 5), the association between starting conditions and organic amendment is strongly significant. (Also, these starting conditions explain an adjusted 5 percent of model variation — strikingly high for a relationship that spans one decade).

Of course, the associations presented in Table 4 are not causal. Even though starting conditions were measured a decade before the dependent variables were measured, a

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[8] At least, this may be true in comparison to the mountainous regions where soil conservation structures are generally studied. Comparatively, our survey areas are quite flat.

combination of omitted relevant variables and autocorrelation over time may bias the estimated coefficients on starting conditions. To gauge the possibility of causal relationships, we therefore control for a host of relevant 2013 variables and examine the stability of the coefficients on soil starting conditions.

Table 5 holds these examinations of coefficient stability for the binary variable indicating any organic amendment being applied to the plot. While column 5 of from Table 4 was estimated without controls, columns 1-4 of Table 5 control for for plot management characteristics, crop dummies, plot size and location characteristics, and 2013 soil fertility. In column 5 all of these characteristics are simultaneously controlled for.

Remarkably, the relationships remain significantly associated with organic amendment across all columns. Column 4 controls for 2013 soil fertility, and though the coefficients on 2003 soil organic carbon and 2003 plot value reduce slightly in magnitude, they are still significantly associated with 2013 organic amendment. In column 5, the set of all controls triples the R^2 of the model from that of column 5 of Table 4, indicating that these are good set of variables for testing coefficient stability (Oster, 2014). It seems possible, therefore, that the associations estimated here are causal, showcasing the importance of soil starting conditions for subsequent organic amendment and soil fertility trajectories. However, without exogenous variation in soil starting conditions — a tall order — we cannot know for sure.

Table 6 examines the 2003 plot conditions associated with 2013 conservation practices, in a similar vein as Table 4. On the whole, these individual practices are not individually, significantly associated with soil staring conditions. However, the indicator variable for any conservation practice being used is significantly associated with soil fertility — in column 5, 2003 soil organic carbon positively predicts conservation practices, and 2003 soil macronutrient content negatively predicts conservation practices.

As with organic amendment, we know of course that these associations are not necessarily causal. Table 7 therefore tests for coefficient stability in the same way that Table 5 does. Columns 1-4 control for the same set of relevant variables, and again the associations between starting conditions and conservation practices is robust to all controls. In fact, in this case the significant coefficients on soil organic carbon and soil macronutrients actually increase in magnitude in column 4, when 2013 soil fertility is controlled for. The coefficients in column 5, where all plot characteristics are controlled for, are statistically identical to the coefficients in column 5 of Table 6.

Last, we examine conservation structures built on the plot. Table 8 examines the associations between 2003 plot conditions and 2013 conservation structures, and finds none that are significant. Even the variable indicating any conservation structure being built is unrelated to 2003 starting conditions.

5 Conclusion

Agricultural and resource economists, agronomists and policy-makers consistently wonder why smallholder farmers don't invest more in soil fertility, either through short-term investments such as compost and manure, or through long-term investments such as rotation, fallowing, or other practices that maintain soil fertility. Additionally, while some researchers have found that soil investment increases with soil fertility (Amsalu and De Graaff, 2007; Yirga and Hassan, 2014), other researchers have found incentives to invest and investment itself decreasing with soil fertility (Jama, Buresh and Place, 1998; Antle, Stoorvogel and Valdivia, 2006; Marenya and Barrett, 2009*b*).

Our model adds nuance to this discussion. Due to the fact that labor is constrained, there exists a trade-off between labor allocated directly to production, labor allocated towards building soil fertility directly through organic amendments (e.g., application of manure or compost) and labor allocated towards practices or structures that mitigate or halt soil degradation (e.g., terracing, soil bunds, fallowing). By examining optimal labor allocation under such a model, we highlight two key messages.

First, it will always be optimal to apply greater levels or organic amendment on lower fertility plots, and less organic amendment on higher fertility plots. This stems from the fact that the marginal return on organic amendment is diminishing with soil fertility, as noted by soil scientists (Dawe et al., 2003; Duncan et al., 2016).

Second, if structural investments are fairly laborious, and/or not very effective at halting soil degradation, they are not optimal. Such structural investments do not directly build soil fertility, and they contribute nothing to current production or income. Indeed, if they take too much labor from the application of organic amendment or from labor allocated directly to production (e.g., weeding, pruning), they may reduce current production and income and not generate compensating future gains, thus proving costly and not worthwhile. Even if they are feasible in terms of labor requirement, they will be optimal only on the higher fertility, higher profit plots. This is because they pay-off associated with structural investments take time to accrue — eventually, soil fertility will reach a higher steady state if soil degradation is mitigated, but the response is not immediate, and depends crucially on continued labor towards organic amendment. Therefore, for plots with low starting soil fertility, the discounted, expected returns on structural investment may be too little to be worthwhile.

Our empirical results are largely in line with these concepts. Organic amendment is associated with soil starting conditions as expected — it appears that over time, farmers apply greater levels of organic amendment to soils that began with low soil organic carbon levels, and apply less organic amendment to soils that began with higher soil organic carbon levels. This relationship is remarkably stable in the face of many, relevant controls, suggesting that the association may be causal.

In addition, it appears that higher-value plots (plots that generate more revenue in 2003) are more likely to receive organic amendment in the subsequent years. This suggests that labor costs associated with organic amendment may be higher than reflected in our model.

The empirical relationship between starting soil fertility conditions and conservation practices/structures designed to mitigate degradation and/or erosion is less clear. To begin with, very few farmers invest in such practices or structures. This suggests that either (i) labor costs (or other costs) associated with such practices/structures are high, (ii) these practices/structures are not effective in halting soil degradation, or (iii) soil degradation is not a significant problem in these contexts. Since such structural investments are often studied in mountainous locations, it may be that soil degradation, or at least soil erosion, is simply not a serious problem in Uganda.

Additionally, while we do find that 2003 soil organic matter is positively associated with conservation practices (in line with our model), we also find that 2003 soil macronutrients are negatively associated with conservation practices (not in line with our model). Conservation structures have no relationship with any 2003 plot conditions.

Our model cleanly separates organic amendment on the one hand, which builds soil fertility but does not mitigate soil degradation, from structural investments on the other hand, which do not build soil fertility but do mitigate soil degradation. Most of the conservation practices/structures that we examine may contribute to soil fertility as well as mitigate soil degradation — and some, like rotation, may also increase production directly. Given this fact, it is perhaps not surprising that the empirical associations between starting soil conditions and these practices/structures is not as clear as the relationship estimated between starting soil conditions and organic amendment.

Additionally, it is worth noting that the soil fertility and soil degradation impacts of various practices may differ greatly across contexts. Our model and our empirical results suggest that soil starting conditions matter to subsequent, optimal soil investment strategies, and therefore to soil fertility trajectories. This will likely be true in many smallholder contexts. The details of these relationships will surely differ across contexts, however, as the relevant dimension of soil fertility and soil degradation change and the impacts of various soil investment practices change.

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Figures

Figure 1: Labor Towards Organic Amendment vs. Structural Investment

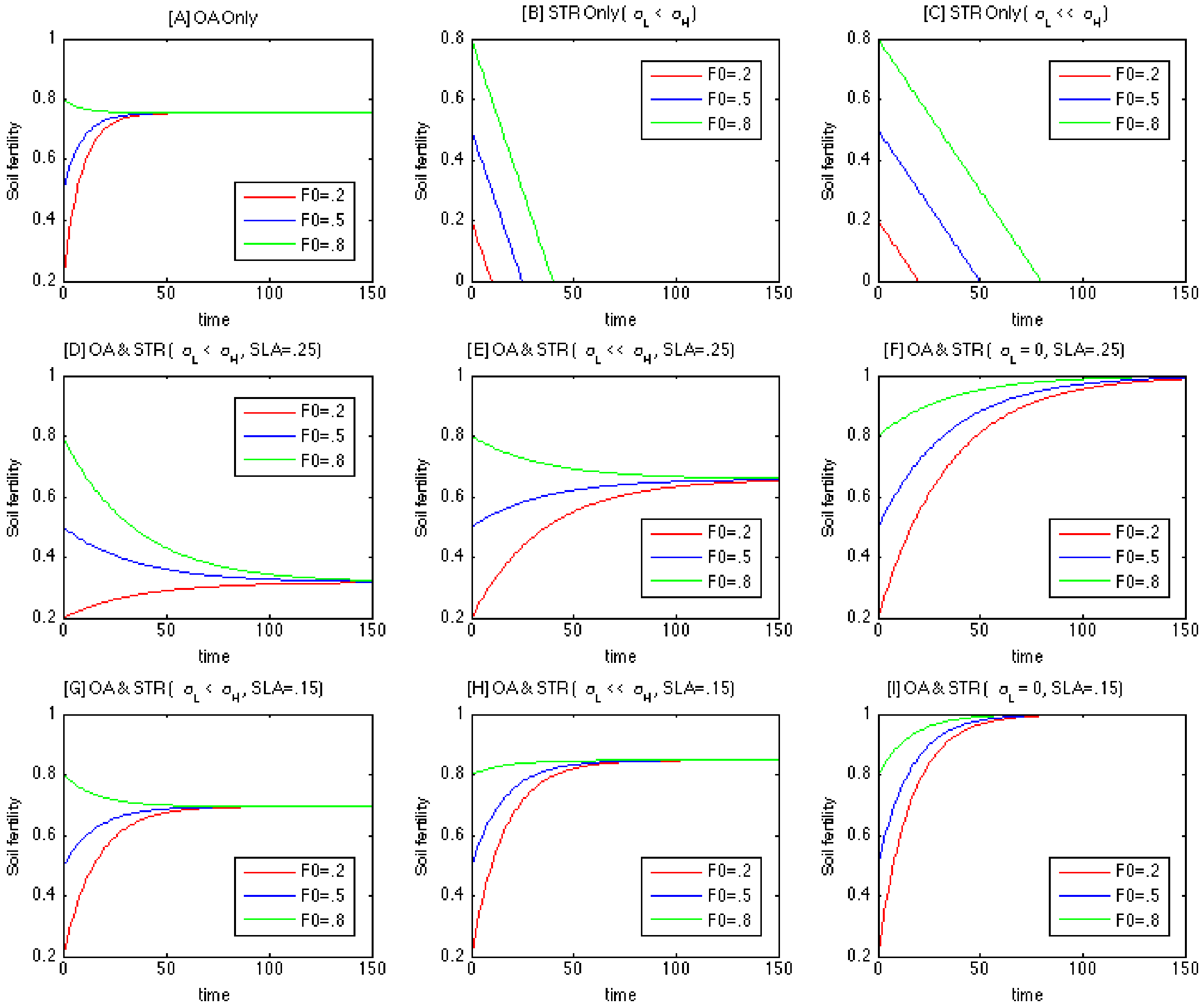


Figure 2: Optimal Fertility, Profit, Steady State Condition $\left(SLA = 0.25, \alpha_L = \frac{2}{3}\alpha_H\right)$

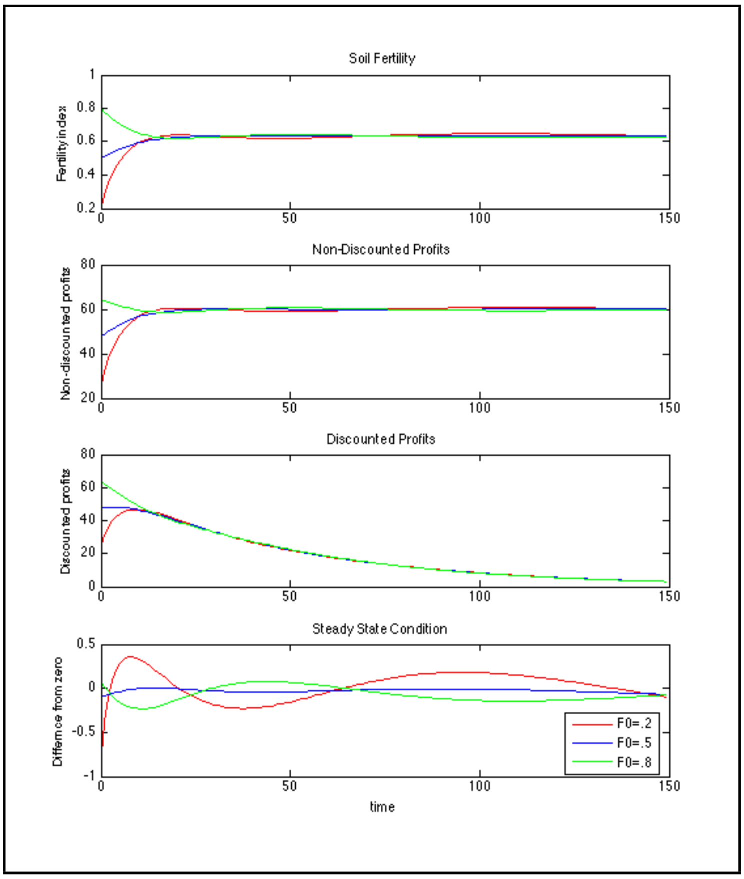


Figure 3: Optimal Labor $\left(SLA = 0.25, \alpha_L = \frac{2}{3}\alpha_H\right)$

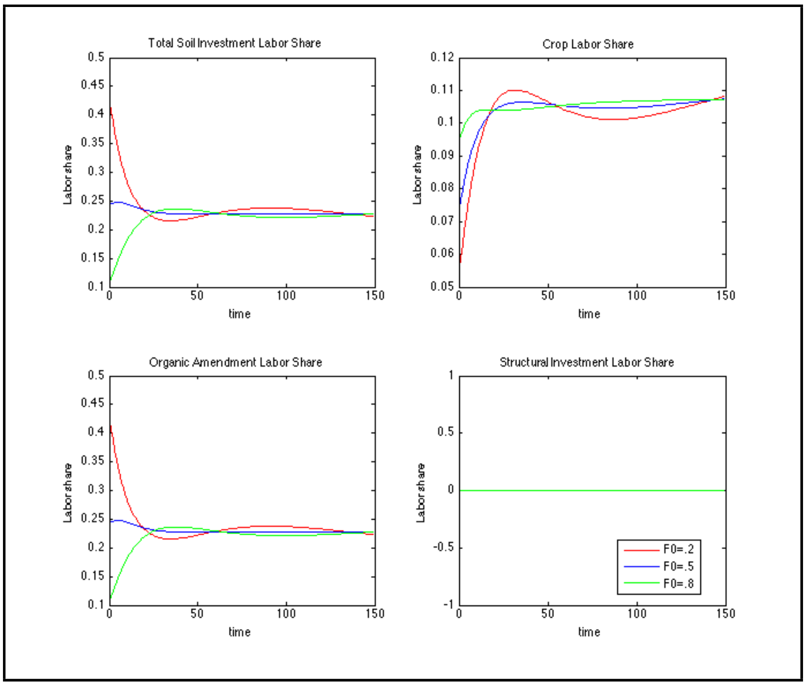
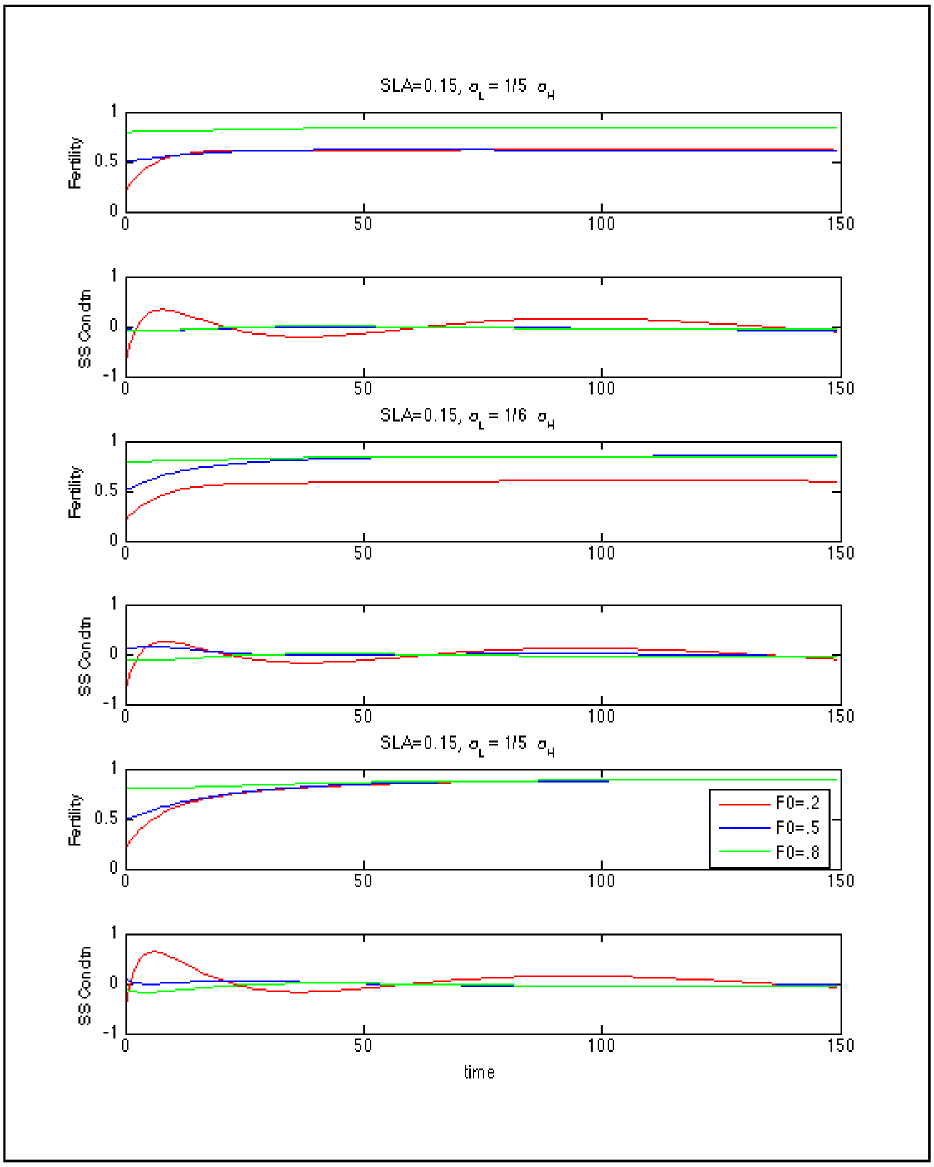


Figure 4: Optimal Fertility, Steady State Condition $\left(SLA = 0.15\right)$



Tables

Table 1: Model Parameters

Model Parameters	Value
δ	0.02
$\rho = \frac{1}{1+\delta}$	0.9804
ω	150
γ_c	0.2
γ_s	0.1
$\gamma_f = 1 - \gamma_c - \gamma_s$	0.7
$\lambda = \frac{2}{ln(2)}$	2.89
L	1
α_H	0.03
α_L	$\in [0, \frac{2}{3}\alpha_H]$
SLA	$\in [0.15, 0.25]$

Table 2: Soil Investment (Plot and Farm-Level Average)

	Plots	Houses
Organic Amendment		
Mulch in last 12 months (%)	0.05	0.06
Household refuse in last 12 months (%)	0.04	0.06
Compost in last 12 months (%)	0.01	0.01
Other manure in last 12 months (%)	0.02	0.03
Cow manure in last 12 months (%)	0.03	0.05
Any organic amendment in last 12 months (%)	0.10	0.15
Conservation Practices		
Generally practices fallowing (%)	0.09	0.12
Generally practices rotation (%)	0.40	0.42
Generally practices mulching (%)	0.14	0.18
Generally practices cover cropping (%)	0.03	0.03
Generally practices no till (%)	0.03	0.03
At least 1 practice used (%)	0.58	0.63
Conservation Structures		
Ridges, strips present (%)	0.06	0.07
Terraces present (%)	0.09	0.12
Drainage ditches present (%)	0.02	0.02
At least 1 structure present (%)	0.16	0.20

Table 3: Plot Characteristics (Plot and Farm-Level Average)

	Plots	Houses
Soil Fertility 2003		
Soil organic carbon '03 (% soil)	3.29	3.25
Soil macronutrients '03 (%N+%P+%K soil)	71.93	70.09
Plot productivity '03 (revenue/ha)	299.67	283.87
Management 2013		
Head owns plot (%)	0.75	0.77
Head manages plot (%)	0.63	0.64
(Head owns)X(Head manages)	0.58	0.59
Crop Dummies 2013		
Tubers grown (%)	0.26	0.36
Cereals grown (%)	0.45	0.61
Legumes grown (%)	0.42	0.58
Bananas grown (%)	0.21	0.29
Cash crops grown (%)	0.17	0.23
Size, Distance 2013		
Distance to house (km)	0.23	0.21
Plot size (ha)	0.32	0.34
Perimeter-area ratio (m/ha)	1.74	1.51
Soil Fertility 2013		
Soil organic carbon '13 (% soil)	3.62	3.57
Soil macronutrients '13 (%N+%P soil)	34.27	33.95
Plot productivity '13 (revenue/ha)	1,253.04	1,010.64

Table 4: Individual Organic Amendments (OLS FE)

	(1) Mulch	(2) Crop Resid	(3) Compost	(4) Manure	(5) Any
Soil organic matter '03 (%)	-0.0175**	-0.0155*	0.00158	-0.0123	-0.0383**
	(0.00837)	(0.00842)	(0.00164)	(0.0110)	(0.0153)
Soil NPK '03 (log %)	-0.0522	0.0330	-0.00490	0.0354	-0.0380
	(0.0435)	(0.0268)	(0.00500)	(0.0483)	(0.0580)
Plot value '03 (log rev/ha)	0.0181*	0.00745	0.00148	0.00208	0.0348***
	(0.0102)	(0.00969)	(0.00151)	(0.00896)	(0.0132)
Observations	715	715	715	715	715
Adjusted R^2	0.025	0.009	-0.001	0.003	0.052

Estimated with household fixed effects
Standard errors clustered by household
*** p<0.01, ** p<0.05, * p<0.1

Table 5: Organic Amendment, All Types Pooled (OLS FE)

	(1) Organic Amend	(2) Organic Amend	(3) Organic Amend	(4) Organic Amend	(5) Organic Amend
Soil organic matter ’03 (%)	-0.0393**	-0.0391**	-0.0344**	-0.0294**	-0.0274**
	(0.0154)	(0.0161)	(0.0151)	(0.0136)	(0.0135)
Soil NPK ’03 (log %)	-0.0428	-0.0572	-0.0528	-0.0232	-0.0499
	(0.0584)	(0.0585)	(0.0608)	(0.0550)	(0.0530)
Plot value ’03 (log rev/ha)	0.0336***	0.0368***	0.0362**	0.0194*	0.0205*
	(0.0129)	(0.0135)	(0.0141)	(0.0111)	(0.0113)
Management 2013	Yes	No	No	No	Yes
Crop dummies 2013	No	Yes	No	No	Yes
Size, distance 2013	No	No	Yes	No	Yes
Soil fertility 2013	No	No	No	Yes	Yes
Observations	715	715	712	625	622
Adjusted R^2	0.0940	0.144	0.0779	0.0373	0.153

Estimated with household fixed effects
Standard errors clustered by household
*** p<0.01, ** p<0.05, * p<0.1

Table 6: Individual Conservation Practices (OLS FE)

	(1) Fallowing	(2) Rotation	(3) Mulching	(4) C-Cropping	(5) Any
Soil organic matter ’03 (%)	0.0179	0.0272	0.00656	0.00901	0.0460**
	(0.0143)	(0.0196)	(0.0149)	(0.00663)	(0.0183)
Soil NPK ’03 (log %)	-0.0814	-0.119*	-0.00894	-0.0539	-0.220***
	(0.0681)	(0.0679)	(0.0495)	(0.0371)	(0.0833)
Plot value ’03 (log rev/ha)	-0.0449***	-0.0238	0.0227	-0.000628	-0.0202
	(0.0152)	(0.0157)	(0.0153)	(0.00224)	(0.0182)
Observations	715	715	715	715	715
Adjusted R^2	0.037	0.019	0.008	0.016	0.038

Estimated with household fixed effects
Standard errors clustered by household
*** p<0.01, ** p<0.05, * p<0.1

Table 7: Conservation Practices, All Types Pooled (OLS FE)

	(1) Consv Ag	(2) Consv Ag	(3) Consv Ag	(4) Consv Ag	(5) Consv Ag
Soil organic matter ’03 (%)	0.0446**	0.0458***	0.0458***	0.0529***	0.0467***
	(0.0184)	(0.0168)	(0.0174)	(0.0196)	(0.0173)
Soil NPK ’03 (log %)	-0.212**	-0.206***	-0.209**	-0.234***	-0.227***
	(0.0870)	(0.0789)	(0.0825)	(0.0833)	(0.0812)
Plot value ’03 (log rev/ha)	-0.0197	-0.0284	-0.0180	-0.0251	-0.0412**
	(0.0184)	(0.0174)	(0.0183)	(0.0205)	(0.0197)
Management 2013	Yes	No	No	No	Yes
Crop dummies 2013	No	Yes	No	No	Yes
Size, distance 2013	No	No	Yes	No	Yes
Soil fertility 2013	No	No	No	Yes	Yes
Observations	715	715	712	625	622
Adjusted R^2	0.0543	0.106	0.0617	0.0746	0.195

Estimated with household fixed effects
Standard errors clustered by household
*** p<0.01, ** p<0.05, * p<0.1

Table 8: Individual Structural Investment (OLS FE)

	(1) Ridges	(2) Terraces	(3) Ditches	(4) Any
Soil organic matter ’03 (%)	-0.00394	-0.00173	-0.000828	-0.00249
	(0.00492)	(0.0125)	(0.000896)	(0.0128)
Soil NPK ’03 (log %)	-0.0256	-0.0600	-0.00542	-0.0766
	(0.0377)	(0.0592)	(0.00552)	(0.0683)
Plot value ’03 (log rev/ha)	-0.00222	-0.00522	-0.00100	-0.00986
	(0.00544)	(0.0159)	(0.00103)	(0.0169)
Observations	715	715	715	715
Adjusted R^2	0.001	0.004	-0.001	0.007

Estimated with household fixed effects
Standard errors clustered by household
*** p<0.01, ** p<0.05, * p<0.1

Appendix 1 The Farmer's Problem: Analytical Steady State Solution

Farmers maximize expected profits over P plots and over infinite time periods. In each time period t, for every plot p, decisions must be made regarding the quantity of labor allocated towards organic amendment to soils, LO_{pt}, and the quantity of labor allocated directly towards crops, LC_{pt}. Those quantities are continuous.

Labor may also be allocated in each time period towards a third activity LS_{pt}, maintaining structural investments in soils. This labor allocation is discrete, such that $LS_{pt} = 0$ or $LS_{pt} = \theta L$, where $0 < \theta < 1$. Additionally, the decision to invest in soils is fixed in time period 0 such that $LS_{pt} = LS_{p0} \forall t$.

An optimal combination of continuous and discrete choice variables cannot be obtained via first order conditions alone. However, by choosing a finite number of plots the problem can be transformed into a finite series of continuous expected profit maximization problems — one for each combination of possible discrete choice variables. Below, we set $P = 3$, giving us 8 sets of possible discrete choice allocations. Optimal allocations of continuous choice variables can be found for each of these 8 sets, and then maximized, expected profit can be compared across each in order to find the optimal discrete choice allocations.

The following four equations define the farmer's three-plot maximization problem for any particular $[\overline{LS_{10}}, \overline{LS_{20}}, \overline{LS_{30}}]$ within the 8 sets of possible discrete choice allocations.

$$\max_{LC_{pt}, LO_{pt}, LS_{p0}, F_{pt}} \pi = \sum_{p} \sum_{t=0}^{t=\infty} \rho^t \{P * G(LC_{pt}, LO_{pt}; F_{pt})\}$$

$$s.t. \;\; F_{p,t+1} = F(F_{pt}, LO_{pt}, \overline{LS_{p0}})$$

$$F_0 = \{F0_1, F0_2, F0_3\}$$

$$L_t = \sum_{p=1}^{p=3} \left\{ LC_{pt} + LO_{pt} + \overline{LS_{p0}} \right\}$$

The discount rate $\rho = (\frac{1}{1+\delta})$. We specify G as a Cobb-Douglas production function with constant returns to scale: $\omega > 0$, $0 < \gamma i < 1$ for each $i \in \{c, s, f\}$, and $\gamma c + \gamma s + \gamma f = 1$.

$$G(LC_{pt}, LO_{pt}; F_{pt}) = \omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f}$$

We specify F as in the following three equations.

$$\alpha_p = \alpha_p^H - \mathbb{1}\{\overline{LS_{p0}} > 0\} \Big(\alpha_p^H - \alpha_p^L \Big)$$

$$\zeta_{pt} = ln\big(1 - (F_{pt} - \alpha_p)\big)$$

$$F_{p,t+1} = 1 - exp\left\{ \zeta_{pt} - \frac{LO_{pt}}{\lambda} \right\}$$

Because α_p is a function of $\overline{LS_{p0}}$, equal to α_p^H if $\overline{LS_{p0}} > 0$ and equal to α_p^L if $\overline{LS_{p0}} = 0$, we denote $\alpha_p(\overline{LS_{p0}})$ as $\overline{\alpha_p}$. The three equations above may therefore be condensed and re-formed as follows, in order to find an expression for $F_{p,t+1} - F_{pt}$.

$$\begin{aligned}
F_{p,t+1} &= 1 - e^{ln\left(1-F_{pt}+\overline{\alpha_p}\right)-\frac{LO_{pt}}{\lambda}} \\
&= 1 - \left(1 - F_{pt} + \overline{\alpha_p}\right)e^{-\frac{LO_{pt}}{\lambda}} \\
F_{p,t+1} - F_{pt} &= \left(1 - F_{pt}\right) - \left(1 - F_{pt} + \overline{\alpha_p}\right)e^{-\frac{LO_{pt}}{\lambda}} \\
F_{p,t+1} - F_{pt} &= \left(1 - F_{pt}\right)\left(1 - e^{-\frac{LO_{pt}}{\lambda}}\right) - \overline{\alpha_p}e^{-\frac{LO_{pt}}{\lambda}}
\end{aligned}$$

For a three plot problem, the Hamiltonian Equation and the Lagrangian may therefore be specified as below, for each combination of $[\overline{LS_{10}}, \overline{LS_{20}}, \overline{LS_{30}}]$ and $[\overline{\alpha_1}, \overline{\alpha_2}, \overline{\alpha_3}]$.

$$\begin{aligned}
H = {} & P\left[\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f}\right] \\
& +\rho\lambda_{p,t+1}\left[\left(1 - F_{pt}\right)\left(1 - e^{-\frac{LO_{pt}}{\lambda}}\right) - (\overline{\alpha_p})e^{-\frac{LO_{pt}}{\lambda}}\right] \\
& +\mu_t\left[1 - \sum_p \{LC_{pt} + LO_{pt} + \overline{LS_{p0}}\}\right] \\
\mathcal{L} = {} & \sum_{p=1}^{p=3}\sum_{t=0}^{t=T}\rho^t\left\{H + \rho\lambda_{p,t+1}(F_{pt} - F_{p,t+1})\right\}
\end{aligned}$$

The following derivatives are the first order conditions (FOCs) defining optimality.

$$\begin{aligned}
\frac{\partial H}{\partial LC_{pt}} &= P\left(\omega\gamma_c LC_{pt}^{\gamma_c-1} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f}\right) - \mu_t = 0 \\
\frac{\partial H}{\partial LO_{pt}} &= P\left(\omega\gamma_o LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o-1} F_{pt}^{\gamma_f}\right) - \mu_t + \lambda_{p,t+1}\left(\frac{1}{\lambda}\right)\left[1 - F_{pt} + \overline{\alpha_p}\right]e^{\frac{-LO_{pt}}{\lambda}} = 0 \\
-\frac{\partial H}{\partial F_{pt}} &= -\left[P(\omega\gamma_f LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_s} F_{pt}^{\gamma_f-1}) - \rho\lambda_{p,t+1}\left(1 - e^{\frac{-LO_{pt}}{\lambda}}\right)\right] = \rho\lambda_{p,t+1} - \lambda_{t,p} \\
\frac{\partial H}{\partial \rho\lambda_{p,t+1}} &= \left[\left(1 - F_{pt}\right)\left(1 - e^{-\frac{LO_{pt}}{\lambda}}\right) - (\overline{\alpha_p})e^{-\frac{LO_{pt}}{\lambda}}\right] = F_{p,t+1} - F_{pt} \\
\frac{\partial H}{\partial \mu_t} &= 1 - \sum_p \{LC_{pt} + LO_{pt} + \overline{LS_{p0}}\} = 0
\end{aligned}$$

Since the first of the FOCs implies that shadow wage $\mu_t = P\left(\omega\gamma_c LC_{pt}^{\gamma_c-1} LO_{pt}^{\gamma_s} F_{pt}^{\gamma_f}\right)$, we can simplify the first order conditions to be captured in only four equations, as below.

$$P\left(\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f}\right)\left(\frac{\gamma_o}{LO_{pt}} - \frac{\gamma_c}{LC_{pt}}\right) + \lambda_{p,t+1}\left(\frac{1}{\lambda}\right)\left[1 - F_{pt} + \overline{\alpha_p}\right]e^{\frac{-LO_{pt}}{\lambda}} = 0$$

$$-\left[P(\omega\gamma_f LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_s} F_{pt}^{\gamma_f-1}) - \rho\lambda_{p,t+1}\big(1 - e^{\frac{-LO_{pt}}{\lambda}}\big)\right] = \rho\lambda_{p,t+1} - \lambda_{t,p}$$

$$\big(1 - F_{pt}\big)\left(1 - e^{-\frac{LO_{pt}}{\lambda}}\right) - (\overline{\alpha_p})e^{-\frac{LO_{pt}}{\lambda}} = F_{p,t+1} - F_{pt}$$

$$1 - \sum_p \{LC_{pt} + LO_{pt} + \overline{LS_{p0}}\} = 0$$

Under the conditions for optimality above, variables evolve over time. Yet at some point a steady state may be reached, after which all variables will become constant for the rest of time. Note that under this steady state, if it exists, $\rho\lambda_{p,t+1} = \rho\lambda_{pt} = \rho\lambda_p$, and recall that $\rho = \frac{1}{1+\delta}$. Under a steady state solution the following will therefore be true.

$$P\big(\omega LC_p^{\gamma_c} LO_p^{\gamma_o} F_p^{\gamma_f}\big)\left(\frac{\gamma_o}{LO_p} - \frac{\gamma_c}{LC_p}\right) + \lambda_p\left(\frac{1}{\lambda}\right)\left[1 - F_p + \overline{\alpha_p}\right]e^{\frac{-LO_p}{\lambda}} = 0$$

$$-\left[P(\omega\gamma_f LC_p^{\gamma_c} LS_p^{\gamma_s} F_p^{\gamma_f-1}) - \rho\lambda_p\big(1 - e^{\frac{-LO_p}{\lambda}}\big)\right] = \rho\lambda_p - \lambda_p = \rho\lambda_p\left(1 - \frac{1}{\rho}\right) = -\rho\lambda_p\delta$$

$$\big(1 - F_p\big)\left(1 - e^{-\frac{LO_p}{\lambda}}\right) - (\overline{\alpha_p})e^{-\frac{LO_p}{\lambda}} = F_p - F_p = 0$$

$$1 - \sum_p \{LC_p + LO_p + \overline{LS_{p0}}\} = 0$$

The conditions can be slightly manipulated, in order to produce the following:

$$P\big(\omega LC_p^{\gamma_c} LO_p^{\gamma_o} F_p^{\gamma_f}\big)\left(\frac{\gamma_c}{LC_p} - \frac{\gamma_o}{LO_p}\right) = \lambda_p\left(\frac{1}{\lambda}\right)\left[1 - F_p + \overline{\alpha_p}\right]e^{\frac{-LO_p}{\lambda}}$$

$$P(\omega\gamma_f LC_p^{\gamma_c} LS_p^{\gamma_s} F_p^{\gamma_f-1}) = \rho\lambda_p\big(\delta + 1 - e^{-\frac{LO_p}{\lambda}}\big)$$

$$\big(1 - F_p\big) = \left[1 - F_p + \overline{\alpha_p}\right]e^{-\frac{LO_p}{\lambda}}$$

$$\sum_p \{LC_p + LO_p + \overline{LS_{p0}}\} = 1$$

By substituting the third condition into the first condition, we may obtain:

$$P\big(\omega LC_p^{\gamma_c} LO_p^{\gamma_o} F_p^{\gamma_f}\big)\left(\frac{\gamma_c}{LC_p} - \frac{\gamma_o}{LO_p}\right) = \lambda_p\left(\frac{1}{\lambda}\right)\big(1 - F_p\big)$$

$$P(\omega\gamma_f LC_p^{\gamma_c} LS_p^{\gamma_s} F_p^{\gamma_f-1}) = \rho\lambda_p\big(\delta + 1 - e^{-\frac{LO_p}{\lambda}}\big)$$

$$\sum_p \{LC_p + LO_p + \overline{LS_{p0}}\} = 1$$

By dividing the first condition by the second, we obtain the final pair of steady state conditions:

$$\frac{F_p}{\gamma_f}\left(\frac{\gamma_c}{LC_p}-\frac{\gamma_o}{LO_p}\right)=\left(\frac{1}{\rho\lambda}\right)\left[\frac{1-F_p}{\delta+1-e^{-\frac{LO_p}{\lambda}}}\right]$$

$$\sum_p\{LC_p+LO_p+\overline{LS_{p0}}\}=1$$

The second condition, above, holds for all time periods t. The first condition holds specifically in steady state. We should observe, therefore, that the following quantity moves towards zero as a steady state is reached.

$$\frac{F_{pt}}{\gamma_f}\left(\frac{\gamma_c}{LC_{pt}}-\frac{\gamma_o}{LO_{pt}}\right)-\left(\frac{1}{\rho\lambda}\right)\left[\frac{1-F_{pt}}{\delta+1-e^{-\frac{LO_{pt}}{\lambda}}}\right]$$

Appendix 2 The Farmer's Problem: Finding Optimal Trajectories

As stated in Appendix 1, we solve for an optimal trajectory in the three plot problem by finding optimal, expected profit maximizing trajectories for LC_{pt} and LO_{pt} for each combination within the possible set of discrete choice allocations — each possible combination of $[\overline{LS_{10}}, \overline{LS_{20}}, \overline{LS_{30}}]$ values. Then we compare expected profit over each of the 8 possible allocations, in order to find the optimal value for each LS_{p0}.

Once $[\overline{LS_{10}}, \overline{LS_{20}}, \overline{LS_{30}}]$ are set, we wish to solve numerically for optimal allocation trajectories of the continuous variables, LC_{pt} and LO_{pt}, and hence the trajectory of soil fertility F_{pt}. This would be impossible if these variables continued to evolve for all time periods t — no numerical algorithm can optimize over infinite values. However, if at some time period T a steady state is reached, such that $\{LC_{pt}, LO_{pt}, F_{pt}\} = \{LC_p, LO_p, F_p\}$ from the steady state condition of Appendix 1, then we can optimize over values $t \in \{0, T\}$ to find trajectories.

So, to restate the same, let us assume that $\{LC_{pt}, LO_{pt}, F_{pt}\}$ vary over time for all $t \in [0, T-1]$, but then take the value of $\{LC_{pt}, LO_{pt}, F_{pt}\}$ for all $t \in [T, \infty]$. We may now re-define the (discrete choice specific) optimization problem of the farmer as follows.

$$\max_{LC_{pt}, LO_{pt}, F_{pt}} \pi = \sum_{p=1}^{p=3} \left\{ \sum_{t=0}^{t=T-1} \rho^t \{P(\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f})\} + \sum_{t=T}^{t=\infty} \rho^t \{P(\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f})\} \right\}$$

$$\begin{aligned} s.t.\;\; & F_{p,t+1} - F_{pt} = \big(1 - F_{pt}\big)\left(1 - e^{-\frac{LO_{pt}}{\lambda}}\right) - \overline{\alpha_p} e^{-\frac{LO_{pt}}{\lambda}} \\ & F_0 = \{F0_1, F0_2, F0_3\} \\ & L_t = \sum_p \left\{LC_{pt} + LO_{pt} + \overline{LS_{p0}}\right\} \end{aligned}$$

The second term within the maximization problem can be simplified, however, in the following way. Because $P(\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_s} F_{pt}^{\gamma_f})\rho^T$ is not time varying, it can be pulled out of the summation. The remaining summation is an infinite series that can be restated as $1/(1-\rho)$. Using $\rho = 1/(1+\delta)$, the equation can be simplified even further.

$$\sum_{t=T}^{t=\infty} \rho^t \{P(\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f})\}$$

$$\begin{aligned} &= P(\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f})\rho^T \sum_{t=0}^{t=\infty} \rho^t = P(\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f})\rho^T \left[\frac{1}{1-\rho}\right] \\ &= P(\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f})\rho^T \left[\frac{1+\delta}{\delta}\right] = P(\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f})\frac{\rho^{(T-1)}}{\delta} \end{aligned}$$

We may therefore re-define our problem as below, where the objective function has been simplified so that only optimal values of $\{LC_{pt}, LO_{pt}, F_{pt}\}$ for $t \in [0, T-1]$ must be chosen. We know that $\{LC_{pt}, LO_{pt}, F_{pt}\} \equiv \{LC_p, LO_p, F_p\}$ from Appendix 1, and thus by definition the updating constraint and the labor constraint will automatically bind for all $t \in [T, \infty]$. Therefore, we may also apply constraints only for time periods $t \in [0, T-1]$.

$$\max_{LC_{pt}, LO_{pt}, F_{pt}} \pi = \sum_p \left\{ P(\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f}) \frac{\rho^{(T-1)}}{\delta} + \sum_{t=0}^{t=T-1} \rho^t \{ P(\omega LC_{pt}^{\gamma_c} LO_{pt}^{\gamma_o} F_{pt}^{\gamma_f}) \} \right\}$$

$$\begin{aligned} &and \;\; s.t. \;\; \forall t \in [0, T-1] \\ &\qquad F_{p,t+1} - F_{pt} = \big(1 - F_{pt}\big)\left(1 - e^{-\frac{LO_{pt}}{\lambda}}\right) - \overline{\alpha_p} e^{-\frac{LO_{pt}}{\lambda}} \\ &\qquad F_0 = \{F0_1, F0_2, F0_3\} \\ &\qquad L_t = \sum_p \left\{ LC_{pt} + LO_{pt} + \overline{LS_{p0}} \right\} \end{aligned}$$

It is this maximization problem above that we use to find optimal trajectories for investment and soil fertility. Three plots are assumed to exist, with starting soil fertility conditions representing a low-fertility plot, a medium-fertility plot, and a high-fertility plot. Optimal labor trajectories are found in matlab, using an algorithm (fminunc) that determines search direction using the BFGS Quasi-Newton method with a cubic line search procedure.

Because searching for optimal values for each labor task, for each time period, and for each plot is infeasible, the dimensionality of the search is reduced drastically by assuming that each labor trajectory is smooth and can be approximated by a high-order basis function. Legendre polynomials therefore provide a finite dimensional approximation for each continuous labor trajectory, and optimization is done over the coefficients of the polynomial function, rather than the values of labor directly. (Legendre polynomials are used because each additional dimension is orthogonal to the previous dimensions, similar to Chebyshev polynomials.)

Then, rather than utilizing formal constraints on the search space for labor allocations, labor allocations LC_{pt} and LO_{pt} are normalized using the softmax function, or the normalized exponential. This ensures that for all time periods t, and each plot p, labor allocation for each task is greater than zero, and that all forms of labor sum to one for all time periods. More details on these processes — the use of Legendre polynomials and the normalization procedure — can be found in Appendix 3.

The soil fertility vectors F_{pt} are created according to F_0 and the state variable updating equation. Expected profit is defined exactly as above, by summing discounted profit over time periods t through $T-1$ and adding that sum to the term that represents profit from time period T onwards. We choose T as 250, because at this point soil fertility and profits appear to have reached a steady state. Matlab scripts available upon request.

Appendix 3 Optimal Labor Trajectories: Basis Functions and Other Computational Details

As stated in Appendix 1 and Appendix 2, we solve for optimal labor trajectories in the three plot problem by first finding optimal, expected profit maximizing trajectories for LC_{pt} and LO_{pt} for each combination within the possible set of discrete choice allocations — each possible combination of $[\overline{LS_{10}}, \overline{LS_{20}}, \overline{LS_{30}}]$ values. Then we compare expected profit over each of the 8 possible allocations, in order to find the optimal value for each LS_{p0}.

Of course, if executed naively, this means we would search for 6 vectors each containing 250 elements, within each of the 8 possible discrete choice allocations. However, there would be far too many unknowns for the estimation process to be robust, and we therefore use basis functions for a finite approximation of the labor trajectories. This requires only the assumption that labor trajectories are smooth, and can be approximated by some type of polynomial. We then optimize over the basis function coefficients, rather than over each individual element of the labor vectors.

The choice of basis function is not particularly significant. In the primary analysis we use a fourth order exponential series as the basis function: $\delta_1[1-e^{(\frac{-t}{\tau})}]+ \delta_2[1-e^{(\frac{-2t}{\tau})}]+ \delta_3[1-e^{(\frac{-3t}{\tau})}]+ \delta_4[1-e^{(\frac{-4t}{\tau})}]$. This is because we know that labor trajectories should asymptote upwards or downwards towards a steady state.

However, the same results will be obtained even if we use Legendre polynomials as our bias functions, as shown in in Appendix 6. Legendre polynomials, similar to Chebyshev polynomials, allow for pathways of any shape, including labor pathways that do not asymptote. But even with this range of possible shapes, optimal trajectories degrade or incline towards a steady state, as the trajectories found under the exponential approximation do.

In order to ensure that all labor allocations fall between 0 and 1, and that labor sums to 1 in all time periods, we normalize labor using the softmax function, or the normalized exponential function. This is a generalization of the logistic function, used to normalize a k-dimensional vector of arbitrary real values to a k-dimensional vector of normalized real values, each ranging between 0 and 1 and summing to 1. The function is given by $SM(x_j) = \frac{e^{x_j}}{\sum\limits_{k=1}^{k=K} e^{x_k}}$.

Each of the six continuous labor vectors are therefore calculated in two steps. First, each vector is determined according to a four-degree exponential series basis function. (Results are robust to using lower- or higher-degree exponential series.) Second, we normalize each labor vector using the softmax function, or the normalized exponential function. These labor allocation vectors are then used to calculate the soil fertility vector, updating soil fertility starting value time period by time period. Labor and soil fertility values are then used to calculate discounted and non-discounted profit in teach time period, as well as to evaluate the steady state condition.

Appendix 4 Soil Sampling and Analysis

In both survey rounds soil sampling was conducted according to standard protocols for in-field, representative soil sampling. Twelve to twenty sub-samples were taken from each plot, with a thin soil probe that reached down to 20 cm. In plots with very hard soil, occasionally an auger or a hoe was used to collect soil samples, rather than a soil probe. In such cases effort was still made to gather soil down to 20 cm.

Sub-samples were taken from randomly distributed locations around the plot, roughly following zig-zag patterns, but avoiding any "odd" patches of ground such as termite mounds or compost piles. (Soil characteristics associated with such patches may be non-representative of the plot.) After mixing all sub-samples together in a bucket, a representative quantity of 500 grams of soil was gathered for subsequent drying, grinding and analysis.

Soil samples were processed and analyzed at Uganda's National Agricultural Laboratory (NARL), in both 2003 and 2013. In each year they were air dried, ground to pass through a 2-mm sieve, and milled using aluminum or stainless steel grinders.

After grinding, soil sub-samples (roughly 0.5 grams) were analyzed for a number of characteristics. Soil pH was determined in a 2.5:1 water to soil suspension, with the pH measured in the soil suspension after a 30-minute equilibration time (Okalebo, Gathua and Woomer, 2002). Soil organic carbon was determined via the Walkely-Black method (Walkley and Black, 1934). While we believe that the buffer pH changed across 2003 and 2013 for this test, round fixed effects should pick up any difference in mean extraction levels due to this methodological shift. Soil texture, including percentage sand, was determined by hydrometer method in both years, after destruction of organic matter with hydrogen peroxide and dispersion with sodium hexametaphosphate (Bouyoucos, 1936; Okalebo, Gathua and Woomer, 2002).

Appendix 5 Full Simulation Results for Figure 4

Figure A1: Optimal Fertility and Profit $\left(SLA = 0.15, \alpha_L = \frac{1}{5}\alpha_H\right)$

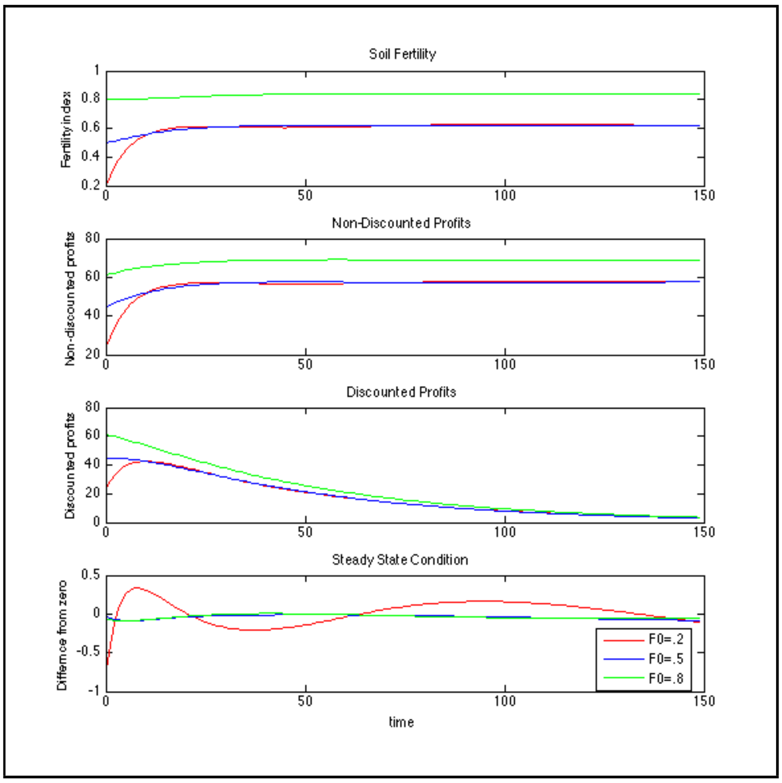


Figure A2: Optimal Fertility and Profit $\left(SLA = 0.15, \alpha_L = \frac{1}{6}\alpha_H\right)$

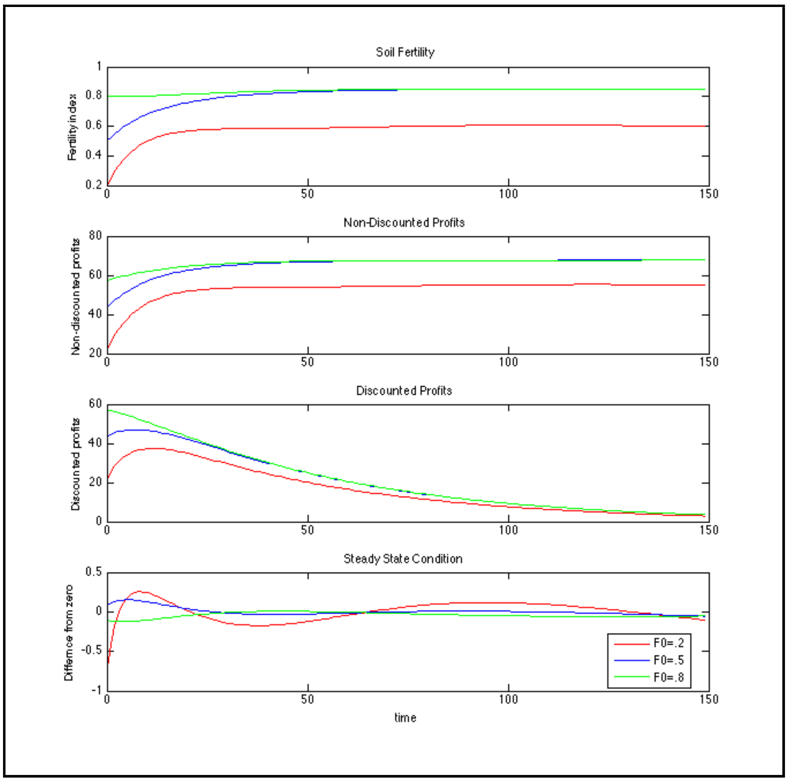
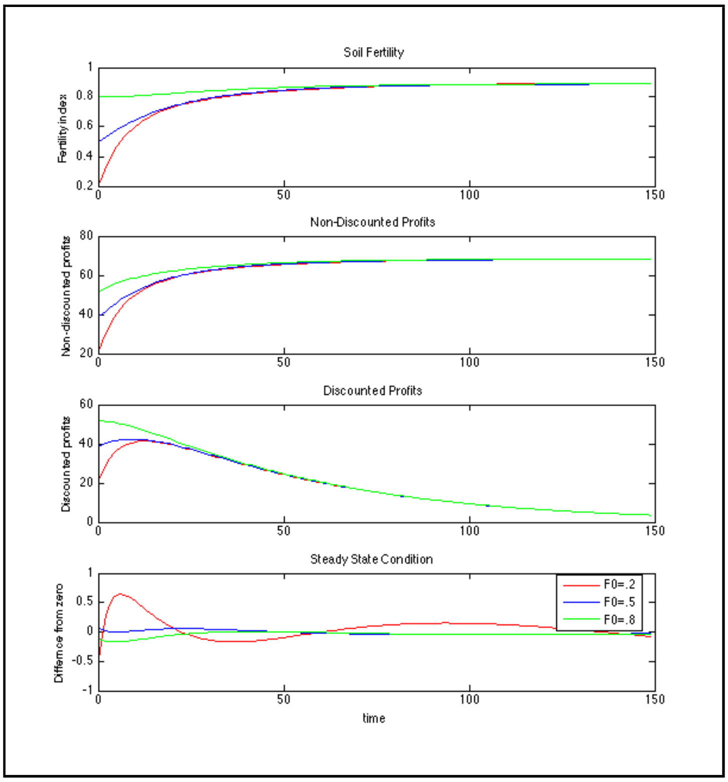


Figure A3: Optimal Fertility and Profit $\left(SLA = 0.15, \alpha_L = \frac{1}{10}\alpha_H\right)$



Appendix 6 Model Results using Legendre Polynomials as Basis Functions

Figure A4: Optimal Fertility and Profit $\left(SLA = 0.25, \alpha_L = \frac{2}{3}\alpha_H\right)$

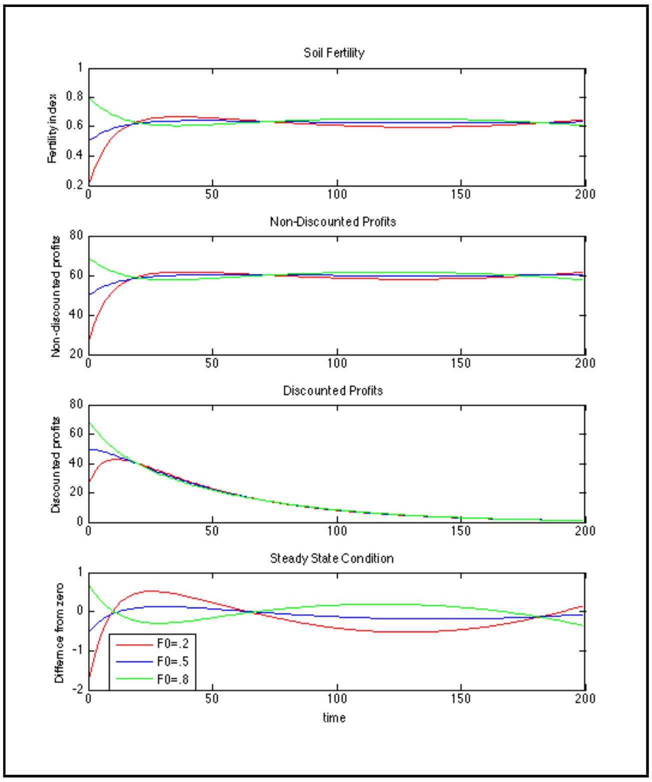


Figure A5: Optimal Labor $\left(SLA = 0.25, \alpha_L = \frac{2}{3}\alpha_H\right)$

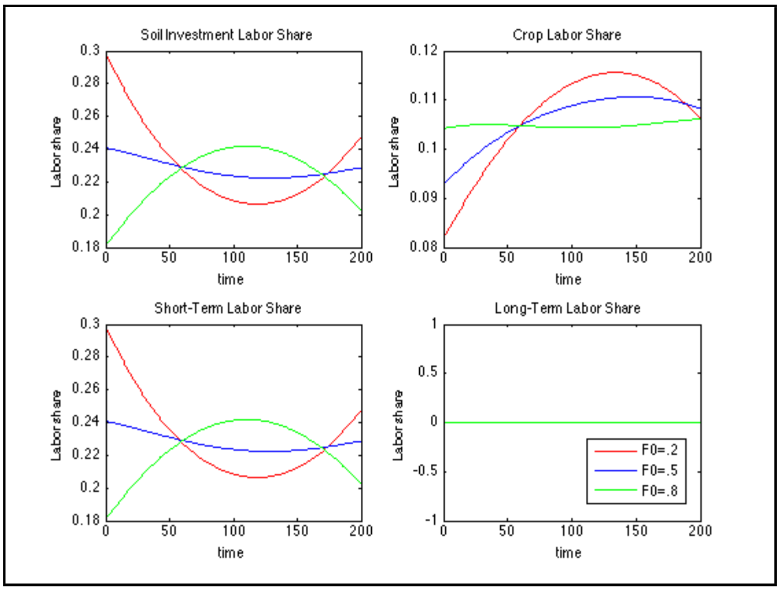


Figure A6: Optimal Fertility and Profit $\left(SLA = 0.15, \alpha_L = \frac{1}{5}\alpha_H\right)$

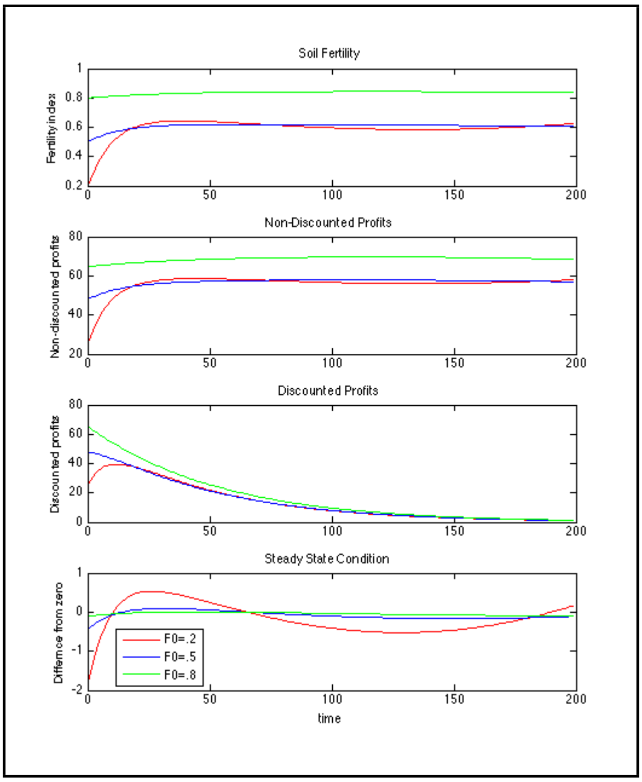


Figure A7: Optimal Fertility and Profit $\left(SLA = 0.15, \alpha_L = \frac{1}{6}\alpha_H\right)$

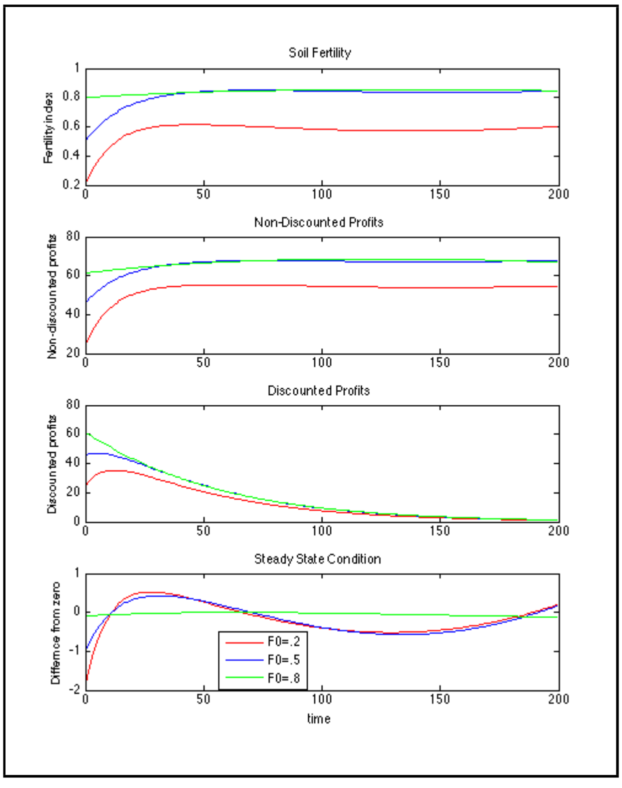


Figure A8: Optimal Fertility and Profit $\left(SLA = 0.15, \alpha_L = \frac{1}{10}\alpha_H\right)$